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Optimizing Product Delivery through Two-Dimensional Time Warping Demand Allocation under Uncertainty

Prita Meilanitasari[1*], Iwan Vanany[2], Mochamad Nizar Palefi Ma'ady [3], Nisa Isrofi[4]

[1]Department Logistic Engineering, Universitas Internasional Semen Indonesia, Indonesia
[2]Department of Industrial and System Engineering, Institut Teknologi Sepuluh Nopember, Indonesia
[3]Department of Information Systems, Institut Teknologi Telkom Surabaya, Indonesia
[4]Department Logistic Engineering, Institut Teknologi Telkom Surabaya, Indonesia

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Corresponding author:
prita.meilanitasari@uisi.ac.id

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Abstract

Purpose – This study aims to optimize delivery operations by implementing a flexible clustering method to handle demand uncertainty and improve logistics efficiency.

Methodology – This study develops a clustering algorithm using a two-dimensional time-warping approach to group demand points based on spatial proximity and demand characteristics. The methodology consists of three stages: 1) processing data on point distances, 2) clustering using two-dimensional time warping, and 3) validating through silhouette analysis.

Findings – This study resulted in optimal and efficient demand clustering through location clustering with a Silhouette coefficient value of 0.7 or an accuracy and feasibility level of 70%. The algorithm also shows improved computational efficiency compared to traditional approaches, making it suitable for practical applications in uncertain and dynamic environments.

Practical implications – This study holds significant importance for businesses in the logistics and retail sectors. Through demand clustering, businesses can effectively group customer demands and utilize this information to optimize inventory management and delivery solutions.

Introduction

Social Medium Enterprises (SME) has become the backbone of Indonesia's economy, contributing to the creation of nearly 107.6 million jobs in Indonesia. Various contributions have been made to economic development, aiming to promote industrialization, balanced income distribution, and job creation (Hidayah, 2021). Several studies on issues related to clusters have proven to have a positive impact. The government has been working to boost the growth of SME, including enhancing research-based technological product innovations for sustainable productivity and fostering collaborative efforts among associations and institutions to strengthen regional innovation systems (Herliana, 2015). Cluster-based systems have also been utilized to prepare SME for international markets by facilitating their readiness to compete internationally (Foghani et al., 2017). The development of SME should not only

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focus on business unit growth but also consider growth factors that affect SME, such as optimizing retail business efficiency.

Currently, the challenge in retail sales demand forecasting is the integration of physical and online orders, which requires extensive data for rapid response (Lalou et al., 2020). Data analysis for demand forecasting plays a crucial role in logistics, particularly in retail distribution network forecasting. There are various stock-keeping units provided by retailers, such as demand frequency, demand variation, demand quantity, and more. The diversity in demand patterns allows for the use of multiple demand forecasting models to achieve high forecast accuracy (Ulrich et al., 2022). Various forecasting techniques are employed in retail industry sales forecasting, but demand patterns can change over time. The use of dynamic models for classifying demand patterns is expected to address the challenge of selecting complex demand models (Erjiang et al., 2022). Categorizing demand patterns is an important issue but is still limited in some literature; the industry tends to categorize demand patterns and estimate future needs while efficiently managing inventory (Kostenko & Hyndman, 2006). Formulating efficient techniques for predicting sales volume is a significant step in supply chain management.

The application of dynamic programming methods for demand clustering in small and medium-sized retail businesses is a crucial aspect of optimizing retail business efficiency. Using this approach, SME can effectively categorize and group demand patterns to enhance operational efficiency and generate revenue. Dynamic Time Warping (DTW) is a standard algorithm for finding optimal alignment between two given time series (Sakoe & Chiba, 1978). Abnormal time series detection is essential in many fields for comparing abnormal time series to a new query time series against a reference (normal) time series. Inferential statistics are used to assess the similarity/distance between two time series in uncertain environments, considering the distance obtained from the DTW algorithm (Duy & Takeuchi, 2023). "Warping" is a computational process that transforms a domain like time into points in time and has varying distances (Pataky et al., 2019).

Literature Review

The conceptualization of SMEs

SME development that encourages industrial development is very important to overcome poverty in the socio-economic sector. Each country has different definitions for SMEs based on different economies and markets. In general, SMEs are independent businesses with a capital investment of less than USD$ 190 million, an annual investment of less than USD$ 70 million, and a number of employees of less than 250 people. In Indonesia, SMEs are classified based on the number of employees. : Micro Enterprises (MIE) (1-4 employees), Small Enterprises (SE) (5-19 employees), SE from Medium Enterprises (ME) (20-99 employees) and medium enterprises from Large Enterprises (LE) (more than 100 employees) (Sofyan, 2019).

Supply chain flexibility

Supply chain and logistics management is faced with several uncertain parameters such as the level of product demand (Sutrisno et al., 2022). It is important to ensure the supply chain remains efficient by maintaining product prices but remaining flexible to changes in demand over time. Uncertainty in the supply chain occurs upstream, process and downstream (Angkiriwang et al., 2014). Technological developments and the large number of competitors require companies to continue to innovate, one of which is by using a flexible supply chain. Global competition leads to competition between companies at the traditional level leading to a level of supply chain efficiency (applying advanced supply chain technology and global sourcing expertise, enabling clients to focus completely on core competencies, lowering sourcing, warehousing, handling and storage costs, reducing working capital and minimizing capital expenditure on distribution assets). Innovation and flexibility in the supply chain increases supply chain resilience in facing uncertain customer demand and production problems due to its ability to share complete information and production planning data (Siagian et al., 2021). By then, logistics flexibility is important for SMEs as it impacts their performance and competitiveness.

Two-Dimensional Time Warping Demand

Clustering is a method for grouping data that has similar characteristics between one data and other data (Yudhanegara et al., 2020). The dynamic programming methodology for demand clustering in small and medium-sized retail businesses offers a sophisticated data-driven solution to further enhance profitability and customer satisfaction, making it a valuable tool for any retail business looking to improve their operations. DTW is an algorithm that nonlinearly aligns two-time series to a set of optimal criteria. Meanwhile, to calculate potential differences in time variations in similar time movements and find similarities between time series, a multidimensional DTW algorithm is used to identify temporal differences between two different time series (Stubinger & Walter, 2022). The implementation of the dynamic programming method for demand clustering in small and medium-sized retail enterprises is a crucial aspect of optimizing retail business efficiency. By utilizing this approach, SMEs can effectively categorize and group demand patterns to enhance operational efficiency and revenue generation. This methodology offers a sophisticated and data-driven solution to further increase profitability and customer satisfaction, making it a valuable tool for any retail business seeking to improve their operations.

An effective demand clustering method for SMEs using dynamic programming can be a game-changer for small and medium-sized enterprises looking to improve their retail operations. By utilizing demand clustering with dynamic programming, businesses can better understand consumer behavior and tailor their products and services accordingly. This can lead to increased sales, customer satisfaction, and overall business success. With this method, SMEs can compete with larger retailers and make a significant impact on their industry.

Research Methods

This study clusters demand locations using two-dimensional time-warping based on node correlation. The research methodology is shown in Figure 1. In Figure 1, the flow of the research methods is shown. The first step involves determining the datasets for node positions, demand capacity, and distance between nodes. Next, the data is processed to clean and sort to identify patterns. The number of clusters is then set based on the customers' target. The method of two-dimensional time-warping is applied to consider flexible demand locations for goods delivery. To ensure the effectiveness of the methods, validation, and verification using Silhouette analysis are included, and the outcome is demanding clustering.

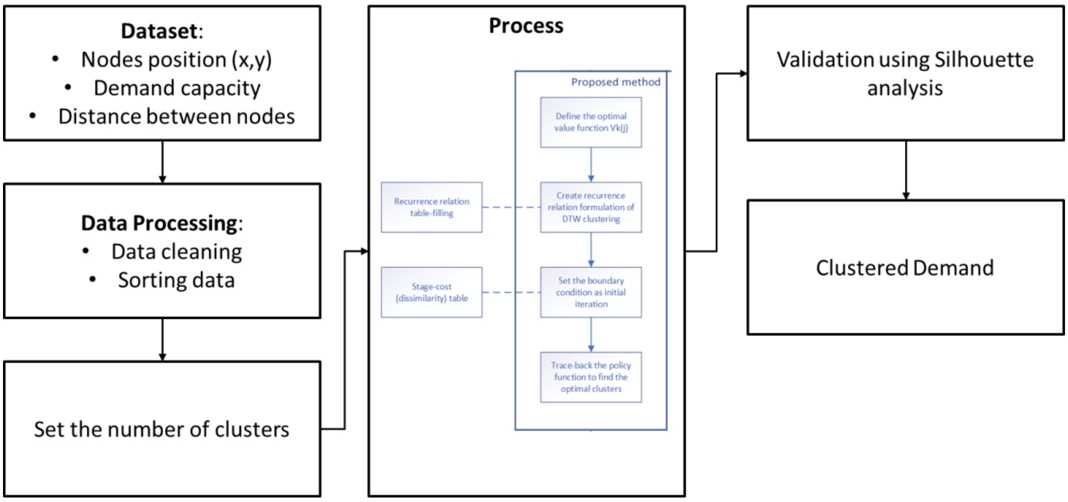

Figure 1. Research Methods

Results and Evaluation

To enhance comprehension, we present a demonstration of our methods. Initially, a stage-cost table is generated using boundary condition (1), which relies on the dissimilarity of product demands that occur consecutively within the sorted data. Subsequently, this stage-cost table facilitates the computation of recurrence relation (2) at a faster pace compared to the numerical approach. As a result, this paper places emphasis on computation time due to the increased efficiency of the table-filling procedure.

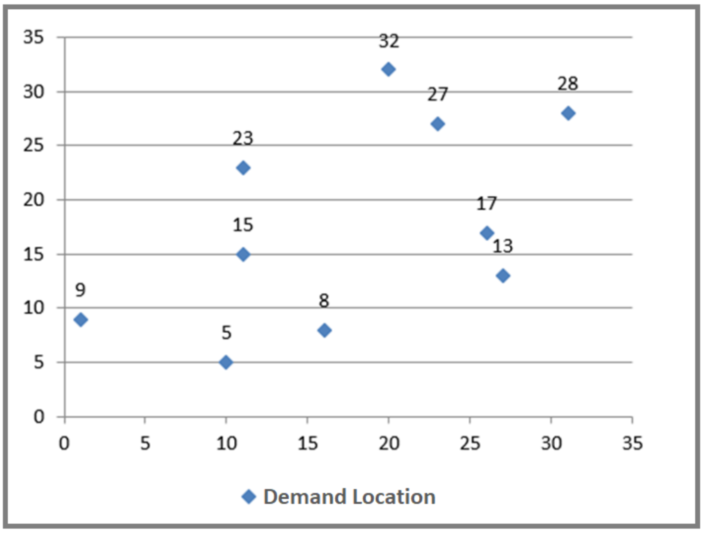

Figure 2. A sample problem of product delivery points (Demand location)

We confine our approach to two-dimensional vectors that are categorized into four clusters denoted as 'c.' Furthermore, we utilize the concept of illegal paths, represented by positive infinity, denoted as $+\infty$, to ensure that unrealistic decisions are not selected. Figure 3 illustrates the y-axis representing 'k' clusters and the x-axis representing 'j' types of products, with the condition that every cluster must contain at least two data point. Based on this information, we can establish that the maximum number of data points in a cluster, denoted as seven data points, is feasible since the total number of data points is ten. To illustrate, within our problem context, if one cluster contains seven data points, the other three clusters should each contain only one data point. Due to the small sample size issue, we manually calculate the distances, as larger datasets can conveniently utilize the Euclidean distance measure.

k										
C_4	$+\infty$	$+\infty$	$+\infty$	8.6	8	18.8	4	10.4	5.8	11.7
C_3	$+\infty$	$+\infty$	6.7	8.6	8	18.8	4	10.4	5.8	$+\infty$
C_2	$+\infty$	9.8	6.7	8.6	8	18.8	4	10.4	$+\infty$	$+\infty$
C_1	0	9.8	6.7	8.6	8	18.8	4	$+\infty$	$+\infty$	$+\infty$
j	(1, 9)	(10, 5)	(16, 8)	(11, 15)	(11, 23)	(27, 13)	(26, 17)	(23, 27)	(20, 32)	(31, 28)

Figure 3. Stage-cost for the sample problem

After applying formula (2), a series of tables are generated, as shown in Figure 3. These tables rely on Figure 4 for calculations and make decisions that move diagonally up. Once the best values are found, we use them to establish the direction of arrows as a policy function. To trace back this function, simply follow the arrows to identify the optimal clusters, which are highlighted in green.

k										
C_4	$+\infty$	$+\infty$	$+\infty$	0	6.7	14.7	18.7	27.3	33.1	42.9
C_3	$+\infty$	$+\infty$	0	6.7	14.7	23.3	27.3	37.1	42.9	$+\infty$
C_2	$+\infty$	0	6.7	15.3	23.3	33.1	37.1	47.5	$+\infty$	$+\infty$
C_1	0	9.8	16.5	25.1	33.1	51.9	55.9	$+\infty$	$+\infty$	$+\infty$
j	P_9	P_5	P_8	P_{15}	P_{23}	P_{13}	P_{17}	P_{27}	P_{32}	P_{28}

Figure 4. Optimal value path provided by the recurrence relation.

Based on Figure 4, the optimal clusters of DTW algorithm is as follows:

$C_4 = P28$

$C_3 = P32, P27$

$C_2 = P17, P13$

$C_1 = P23, P15, P8, P5, P9$

To begin, we generate a dissimilarity matrix using the dataset, as illustrated in Table 1. This matrix results in a symmetric matrix with zeroes along the diagonal. Subsequently, we calculate and as presented in the following tables.

Tabel 5. Dissimilarity matrix for the dataset.

Data	**(1,9)**	**(10,5)**	**(16,8)**	**(11,15)**	**(11,23)**	**(27,13)**	**(26,17)**	**(23,27)**	**(20,32)**	**(31,28)**
(1,9)	0	9,8	15	11,6	17,2	26,3	26,2	28,4	29,8	35,5
(10,5)	9,8	0	6,7	10	19	18,7	20	28	28,8	31
(16,8)	15	6,7	0	8,6	15,8	12	13,4	20	24,3	25
(11,15)	11,6	10	8,6	0	8	16	15	17	19	23,8
(11,23)	17,2	19	15,8	8	0	18,8	16	12,6	12,7	20,6
(27,13)	26,3	18,7	12	16	18,8	0	4	14,5	20	15,5
(26,17)	26,2	20	13,4	15	16	4	0	10,4	16	12
(23,27)	28,4	28	20	17	12,6	14,5	10,4	0	5,8	8
(20,32)	29,8	28,8	24,3	19	12,7	20	16	5,8	0	11,7
(31,28)	35,5	31	24	23,8	20,6	15,5	12	8	11,7	0

By Table 2, we analyze the distribution of Silhouette Coefficients (SC) of the proposed method. The Silhouette Coefficient for each data point is provided in Table 2 below.

Tabel 2. Silhouette Coefficient on each data for the proposed method

Data	Proposed method		
	a	*b*	SC
(1,9)	13.40	29.24	0.542
(10,5)	11.37	25.30	0.551
(16,8)	11.52	18.94	0.392
(11,15)	9.55	18.16	0.475
(11,23)	15	16.14	0.071
(27,13)	4	17.73	0.775

(26,17)	4	16.13	0.753
(23,27)	5.8	17.36	0.666
(20,32)	5.8	20.29	0.715
(31,28)	1	20.34	0.951

Once we have processed each data point, we proceed to calculate the Silhouette Coefficient (SC) for each cluster using the information from Table 2. Table 3 presents the SC values obtained by both methods for each cluster.

Tabel 3. Silhouette Coefficient for each cluster

Cluster	Proposed method
1	0.4062
2	0.764
3	0.691
4	0.951

Comparing the results, as indicated in Table 3, it becomes evident that the two-dimensional time warping approach achieves a high Silhouette score. This signifies that the grouping method's effectiveness is more pronounced, with scores of 0.703. Additionally, it's worth noting the faster computational performance of dynamic time warping, with a time complexity which has a time complexity of $O(m * n)$.

Conclusion

For businesses in the logistics and retail sectors especially, accurate demand forecasting is of paramount importance due to the significant impact it has on financial aspects. The sustainability of a retailer's business hinges greatly on its financial performance. Given the diverse nature of products in a warehouse, which can be classified as non-moving, slow-moving, medium-moving, or fast-moving, it becomes crucial to cluster these products effectively. Therefore, there is a need for an optimized clustering method within the realm of machine learning to handle this task.

In this paper, we propose an optimization method in pattern recognition, namely dynamic time warping, for addressing clustering tasks in this context. Our results demonstrate that dynamic time warping performs with Silhouette scores of 0.703. This suggests that our proposed method is more efficient. Furthermore, it boasts fast with time complexities of $O(m * n)$.

We have also included a numerical demonstration of our approach in the table-filling procedure. In future studies, we are intrigued by the prospect of addressing more complex, n-dimensional clustering problems, as our current focus is limited to two dimensions. Rather than considering only two variables, such as product demand, we aim to explore additional variables like fees, categories, and segmentation in our analysis.

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