

# The Solution of The Richards Equation by The Dual Reciprocity Method for Analyzing Infiltration with The Depth Variation of Trapezoidal Channels in Heterogeneous Soil\*

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**Abstract**—The research discusses the effect of depth variations on the infiltration process of trapezoidal irrigation channels in two layers of heterogeneous soils. There are five depth variations at 2.5 cm, 7.5 cm, 12.5 cm, 17.5 cm, and 22.5 cm. The infiltration problem of this research is constructed in Richards equation as a mathematical model. The model is transformed into the modified Helmholtz equation to get linear partial differential equation. Based on the model, the flux value and boundary conditions are defined and determined based on various assumptions. The model was then solved numerically using the dual reciprocity method (DRM). The numerical solution obtained is the suction potential for each variation in channel depth. The solution indicates that the area below the channels has the highest suction potential. The result shows that the suction potential at the shallowest depths is the lowest. This indicates that the deeper channel produces a higher flux compared to a shallower channel. The solutions also imply that there are jumps in suction potential value from layer 1 to layer 2.

## I. INTRODUCTION

THE dual reciprocity method (DRM) is a numerical method developed from the boundary element method (BEM). The main idea of BEM is that the solution of a partial differential equation (PDE) is expressed in a boundary integral equation involving the fundamental solution of the corresponding PDE. However, not all PDEs have fundamental solutions; one such example is the Helmholtz equation. Consequently, the DRM was developed as an extension of BEM to solve PDEs for which it is difficult to find a fundamental solution.

Previous studies discussing DRM have been conducted by several researchers. Pramesti et al. [1] discussed the application of DRM to heat conduction problems in anisotropic materials. Hamzehei-Javaran et al. [2] examined the application of DRM to solve 2D electrostatic and dynamic problems reformulated using a new complex Fourier shape function.

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Solekhuudin et al. [7] investigated the application of DRM to solve infiltration problems in two-layer soils.

Based on previous research, the study of infiltration problems is particularly interesting. This is because infiltration is a critical process that affects water availability for plants in everyday life. According to Saputra [5], meeting water requirements for crops in the agricultural sector is achieved by creating irrigation infrastructure. One commonly used system by farmers is surface irrigation with the furrow irrigation method [8]. An important aspect of the furrow irrigation system is the infiltration process. Water infiltration is influenced by the type, characteristics, conditions, and content of each soil, as each soil has different constituent substances. Mathematically, water infiltration in soil can be expressed by Darcy's Law as follows:

$$q = -K \nabla H, \quad (1)$$

where  $q$  is the water flux,  $K$  is the unsaturated soil conductivity, and  $\nabla H$  is the hydraulic head gradient.

Equation (1) can be developed into the Richards Equation, which describes the movement of water in saturated soil. The Richards Equation is a nonlinear two-dimensional partial differential equation (PDE) with the general form:

$$\frac{\partial}{\partial X} \left( K \frac{\partial \psi}{\partial X} \right) + \frac{\partial}{\partial Z} \left( K \frac{\partial \psi}{\partial X} \right) + \frac{\partial K}{\partial Z} = 0, \quad (2)$$

where  $\psi$  is the suction potential.

Previous research has mainly focused on time-dependent infiltration from trapezoidal channels in homogeneous soils [11] or on infiltration from trapezoidal channels in layered soils with a fixed channel geometry [9], [10]. Consequently, the effect of varying the depth of the trapezoidal irrigation channel on the resulting heterogeneous soils has not been sufficiently investigated within a DRM framework.

This research aims to investigate Equation (2), the effect of irrigation channel depth variation on infiltration in heterogeneous two-layer soil, solved by the modified Helmholtz equation with the dual reciprocity method. The infiltration model used is the modified Helmholtz equation with mixed boundary conditions, which will be solved using DRM. The solution obtained is the value of suction potential, allowing for

an analysis of the influence of channel depth variations on infiltration in heterogeneous two-layer soils and for determining the appropriate depth of the irrigation channel.

## II. RESEARCH METHOD

### A. Problem Formulation

This study discusses the issue of infiltration in irrigation channels with a two-layer heterogeneous soil structure, where the irrigation channels used have trapezoidal shape. The illustration of the problem in this study can be geometrically described as Fig. 1. The illustration in Fig. 1 shows the occurrence of infiltration in a trapezoidal-shaped irrigation channel. The depth of the channel is represented by the variable  $r$ , and the width of the channel is represented by the variable  $s$ . The detailed data related to channel dimensions of  $r$  are shown in Table I.

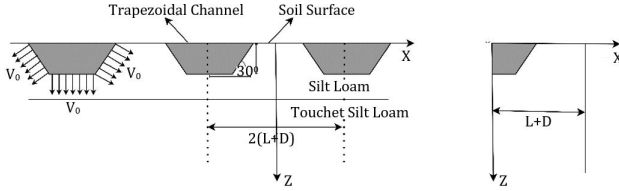


Fig. 1. Geometry of heterogeneous infiltration on trapezoidal channel

Each channel has the same width, which is  $2L$ . The distance between the midpoints of two adjacent channels is  $2(L+D)$ . In this study,  $L$  and  $D$  are chosen to have the same length, namely  $L = D = 50$  cm. Each irrigation channel is very long, and its geometric shape does not change along the  $Y$  axis, so the  $Y$  axis length can be disregarded. Each irrigation channel has a symmetric region with boundaries  $0 \leq X \leq L+D$  and  $Z \geq 0$ . These boundaries are used only to observe the infiltration process. The flux or flow rate entering the surface of the irrigation channel is  $V_0$  assumed to be constant. The saturated hydraulic conductivity of the soil in this study is chosen such that  $V_0 = 0.75K_0$ . The angle between the horizontal line and the sloped surface of the channel is assumed to remain constant at  $30^\circ$ .

TABLE I  
CHANNEL SIZE-RELATED RESEARCH DATA

| Channel Type | Channel Depth ( $r$ ) |
|--------------|-----------------------|
| Channel A    | 2.5 cm                |
| Channel B    | 7.5 cm                |
| Channel C    | 12.5 cm               |
| Channel D    | 17.5 cm               |
| Channel E    | 22.5 cm               |

### B. Research Procedure

The research procedure consists of two main stages: the numerical solution of the infiltration model and the analysis of the effect of trapezoidal channel depth on infiltration in a two-layer heterogeneous soil.

1) *Numerical Solution of the Infiltration Model*: The infiltration model is formulated from Darcy's law and developed into the nonlinear Richards equation. The equation is transformed into a linear modified Helmholtz equation using the Kirchhoff, dimensionless-variable, and exponential transformations as following [11]

#### a. Kirchoff Transformation

$$\Theta = \int_{-\infty}^{\psi} K(s)ds, \quad (3)$$

with  $\Theta$  as the Matric Flux Potential (MFP).

#### b. Exponential Model of Hydraulic Conductivity

$$K = K_0^* e^{\alpha^* \psi}, \quad \alpha^* > 0, \quad (4)$$

with  $\alpha^* = \bar{\alpha}_i$  as the soil parameter of heterogeneous soil and  $K_0^* = \bar{K}_{0i}$  for  $i = 1, 2$  as the saturated hydraulic conductivity of heterogeneous soil.

#### c. Dimensionless Variables

$$\begin{aligned} x &= \frac{\alpha^*}{2} X, \quad z = \frac{\alpha^*}{2} Z, \quad \Phi = \frac{\pi \Theta}{v_0 L}, \\ u &= \frac{2\pi}{v_0 \alpha^* L} U, \quad v = \frac{2\pi}{v_0 \alpha^* L} V, \\ f &= \frac{2\pi}{v_0 \alpha^* L}, \end{aligned} \quad (5)$$

where  $v_0$  is the initial flux and  $L$  is half the length of the irrigation channel.

#### d. Exponential Transformation

$$\Phi = \phi e^z. \quad (6)$$

The final result of the transformation (3)-(6) is the modified Helmholtz equation (7) as follows:

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = \phi. \quad (7)$$

The flux value is obtained as follows:

$$f = -e^z \left( \frac{\partial \phi}{\partial n} - \phi n_2 \right). \quad (8)$$

The normal derivative obtained is:

$$\frac{\partial \phi}{\partial n} = \phi n_2 - e^{-z} f. \quad (9)$$

The derivation of the boundary conditions follows the procedure provided in equation (3)-(6) The resulting boundary conditions for the irrigation channel are given as follows:

$$\begin{aligned} \frac{\partial \phi}{\partial n} &= \frac{2\pi}{\alpha L} e^{-z} + \phi n_2, & \text{on the channel surface,} \\ \frac{\partial \phi}{\partial n} &= -\phi, & \text{for } \frac{s\alpha}{2} \leq x \leq \frac{\alpha}{2}(L+D) \text{ and } z = 0, \\ \frac{\partial \phi}{\partial n} &= 0, & \text{for } x = \frac{\alpha}{2}(L+D) \text{ and } z \geq 0, \\ \frac{\partial \phi}{\partial n} &= 0, & \text{for } x = 0 \text{ and } z \geq \frac{r\alpha}{2}, \\ \frac{\partial \phi}{\partial n} &= -\phi, & \text{for } 0 \leq x \leq L+D \text{ and } z = c. \end{aligned}$$

The resulting equation is subsequently solved using the Dual Reciprocity Method (DRM) as in Section III. The numerical implementation is performed using MATLAB R2015b. The

numerical results are analyzed across the two soil layers and validated against theoretical considerations and previously reported results to assess the accuracy of the proposed DRM formulation.

2) *Effect of Trapezoidal-Channel Depth*: The effect of trapezoidal-channel depth is investigated by applying the developed DRM formulation to several channel-depth configurations. For each configuration, the suction-potential distribution is computed and visualized. The results are compared at selected vertical observation depths of  $z_1 = 2.5$  cm,  $z_2 = 7.5$  cm,  $z_3 = 12.5$  cm,  $z_4 = 17.5$  cm, and  $z_5 = 22.5$  cm to identify the influence of channel depth on the infiltration pattern. The numerical results are also compared with previous studies for validation, and conclusions are drawn based on the observed variation in suction potential.

### III. DUAL RECIPROCITY METHOD

The value of  $\phi$  is the result of the MFP (Matric Flux Potential) transformation, which describes the distribution of water flow and can be used to determine the suction potential in a two-layer heterogeneous soil system, transitioning from fine to coarse texture.

According to [14], suction potential is the suction force of unsaturated soil and has a negative value. The suction potential  $\psi$  can be obtained by the following equation:

$$\psi = \frac{1}{\alpha_i} \ln \left( \frac{\alpha_i v_0 L \phi e^z}{\pi K_{0i}} \right). \quad (10)$$

Alternatively, it can be written as:

$$\psi = \begin{cases} \frac{1}{\alpha_1} \ln \left( \frac{0.75 \alpha_1 L \phi e^z}{\pi} \right), & \text{in layer 1.} \\ \frac{1}{\alpha_2} \ln \left( \frac{0.75 K_1 \alpha_2 L \phi e^z}{\pi K_2} \right), & \text{in layer 2.} \end{cases} \quad (11)$$

Next, the illustration of the boundary conditions of a trapezoidal irrigation channel with curved line segments is as follows:

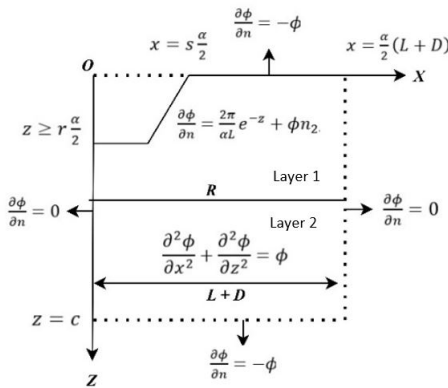


Fig. 2. Geometry of heterogeneous infiltration on Trapezoidal Channel

In this study, the Dual Reciprocity Method (DRM) is applied to compute numerical solutions of Equation (7) under the boundary conditions as in Fig. 2. The method was originally developed by Brebbia and Nardini [4]

The solution of the boundary condition infiltration problem following the DRM (Dual Reciprocity Method) procedure by

substituting the boundary conditions with modified Helmholtz Equation (7) as in Zahroh [17] and [18].

In the present study,  $N$  denotes the number of collocation points located on the boundary of the computational domain, whereas  $M$  denotes the number of collocation points located inside the domain (interior points). Thus, the total number of collocation points used in the Dual Reciprocity Method (DRM) formulation is  $N + M$ .

$$\lambda(a^{(m)}, b^{(m)}) \phi^{(m)} = \sum_{k=1}^N \phi^{(k)} \left[ F_2^{(k)}(a^{(m)}, b^{(m)}) - v^{(k)} F_1^{(k)}(a^{(m)}, b^{(m)}) \right] - \sum_{k=1}^N f^{(k)} e^{-b^{(k)}} F_1^{(k)}(a^{(m)}, b^{(m)}) + \sum_{j=1}^{N+M} \mu^{(m,j)} \phi^{(j)}, \quad (12)$$

for  $m = 1, 2, \dots, N + M$  and

$$\lambda(a^{(m)}, b^{(m)}) = \begin{cases} 0, & \text{if } (a^{(m)}, b^{(m)}) \text{ not in } R \cup C \\ \frac{1}{2}, & \text{if } (a^{(m)}, b^{(m)}) \text{ in } C \\ 1, & \text{if } (a^{(m)}, b^{(m)}) \text{ in } R, \end{cases} \quad (13)$$

$$v^{(k)} = \begin{cases} 0, & C^{(k)} \text{ for horizontal curve} \\ n_2, & C^{(k)} \text{ on the surface of the channels,} \\ 1, & C^{(k)} \text{ the others} \end{cases} \quad (14)$$

$$F_1^{(k)} = \frac{1}{4\pi} \int_{C^{(k)}} \ln((x-a)^2 + (y-b)^2) ds \quad (15)$$

$$F_2^{(k)} = \frac{1}{4\pi} \int_{C^{(k)}} \frac{\partial}{\partial n} [\ln((x-a)^2 + (y-b)^2)] ds \quad (16)$$

where  $R$  is the region and  $C$  is the closed curve that bounds  $R$  (boundary curve).

Let  $\phi^{(m)}$  be the value of  $\phi$  evaluated at  $(a^{(m)}, b^{(m)})$ , and let  $f^{(k)}$  be the incoming flux at the collocation point  $(a^{(k)}, b^{(k)})$ . The terms in (12) that contain  $\phi^{(k)}$ , for  $k = 1, 2, \dots, N + 1, \dots, N + M$ , are collected on the left-hand side, and the remaining terms are moved to the right-hand side, yielding:

$$\sum_{k=1}^N \left[ F_2^{(k)}(a^{(m)}, b^{(m)}) - v^{(k)} F_1^{(k)}(a^{(m)}, b^{(m)}) \right] \phi^{(k)} + \sum_{j=1}^{N+M} \mu^{(m,j)} \phi^{(j)} - \lambda(a^{(m)}, b^{(m)}) \phi^{(m)} = \sum_{k=1}^N f^{(k)} e^{-b^{(k)}} F_1^{(k)}(a^{(m)}, b^{(m)}). \quad (17)$$

Equation (17) can be transformed into a system of linear equations as follows:

$$\sum_{k=1}^{N+M} A_{mk} \phi^{(k)} = b_m, \quad (18)$$

with

$$\begin{aligned}
 b_m &= \sum_{k=1}^N \mathfrak{f}^{(k)} e^{-b^{(k)}} F_1^{(k)}(a^{(m)}, b^{(m)}) \\
 &\text{for } m = 1, 2, \dots, N + M, \\
 A_{mk} &= F_2^{(k)}(a^{(m)}, b^{(m)}) - v^{(k)} F_1^{(k)}(a^{(m)}, b^{(m)}) \\
 &\quad + \mu(a^{(m)}, b^{(m)}; a^{(k)}, b^{(k)}) - \frac{1}{2} \delta^{(mk)} \\
 &\text{for } m = 1, 2, \dots, N, \\
 A_{mk} &= \mu(a^{(m)}, b^{(m)}; a^{(k)}, b^{(k)}) - \delta^{(nk)} \\
 &\text{for } m = 1, 2, \dots, N + M,
 \end{aligned}$$

where the delta is defined as:

$$\delta^{(mk)} = \begin{cases} 0, & \text{if } k \neq m \\ 1, & \text{if } k = m. \end{cases} \quad (19)$$

This scheme is iterated repeatedly until the desired time interval is reached and a certain convergence criterion is met. The numerical solution for the Helmholtz Equation using DRM is computed using a computer program developed in MATLAB. The numerical result obtained is the suction potential ( $\psi$ ). This  $\psi$  value must be validated by comparing it with previous studies. The depth  $r$  modifies the boundary point coordinates  $\{(a^{(k)}, b^{(k)})\}$  on the base and the two sloping sides of the trapezoid. The normal vector  $n^{(k)}$  and the element length between collocation points also change. The final step in the research is result analysis based on the MATLAB output after validation, followed by drawing conclusions based on the analysis.

#### IV. EXPERIMENTAL RESULTS

The numerical results were obtained to solve the problem of the effect of depth variations on the infiltration process of trapezoidal irrigation channels in two layers of heterogeneous soils. There are five depth variations at channel A, channel B, channel C, channel D, channel E as in Table I. The research data related to soil properties is presented in Table II.

TABLE II  
SOIL-RELATED RESEARCH DATA

| Soil Type         | $\alpha$ (cm <sup>-1</sup> ) | $K_0$ (cm/day) |
|-------------------|------------------------------|----------------|
| Silt Loam         | $1.39 \times 10^{-2}$        | 4.9594         |
| Touchet Silt Loam | $1.56 \times 10^{-2}$        | 41.99          |

The soil proportion parameter ( $\alpha$ ) and hydraulic conductivity ( $K_0$ ) for Touchet Silt Loam were obtained from literature [16]. The values for Silt Loam were sourced from literature [7]. Meanwhile, the channel depth and other assumptions related to channel dimensions are adapted from reference [6]. The DRM solution uses discretization based on the boundary points ( $N$ ) and interior points ( $M$ ). The discretization details for each depth are presented in Table III below:

The values of  $\phi^{(k)}$  and  $p^{(k)}$  are obtained using the DRM and are subsequently used for evaluation at several locations within the domain. The next step is performed using a MATLAB program. The final result obtained is the value of the

TABLE III  
DISCRETIZATION DETAILS FOR EACH CHANNEL DEPTH

| Channel Type | ( $N$ ) | ( $M$ ) |
|--------------|---------|---------|
| Channel A    | 170     | 400     |
| Channel B    | 168     | 392     |
| Channel C    | 166     | 388     |
| Channel D    | 164     | 385     |
| Channel E    | 161     | 384     |

suction potential ( $\psi$ ) by using Equation (11). The infiltration characteristics at each channel depth based on the plot results are shown in Fig. 3.

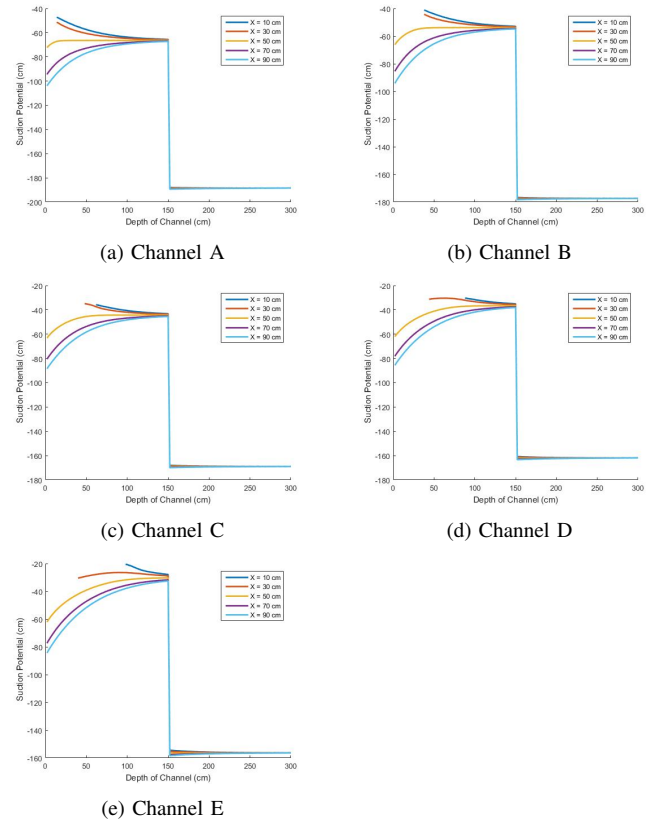


Fig. 3. Graphs of suction potential for 5 types of channel selected values of  $X$  along  $Z$ -axis under impermeable condition

Fig. 3 shows the suction potential values ( $\psi$ ) in a trapezoidal irrigation channel with five depth variations: channel A, channel B, channel C, channel D, channel E, along  $X = 10$  cm,  $X = 30$  cm,  $X = 50$  cm,  $X = 70$  cm, and  $X = 90$  cm.

The curves at  $X = 10$  and  $X = 30$  in Fig. 3 and 4 do not start from  $z = 0$  because the upper parts of these vertical observation lines are located outside the soil domain and within the irrigation-channel geometry. This occurs because those positions intersect the irrigation-channel region before entering the soil domain. Since infiltration is evaluated only within the soil domain, the corresponding suction-potential values are available only below the channel boundary.

Suction potential (sometimes called matrix potential or soil water potential) describes the energy state of water in soil due to capillary and absorptive forces. It is expressed as a negative pressure (tension), because water in soil is usually

under suction compared to free water at atmospheric pressure. Upon examining the plot, each channel depth variation exhibits similar suction potential distribution characteristics, suggesting that the analysis results may be applicable to other channels as well. In general condition, A value of  $-20 \text{ cmH}_2\text{O}$  ( $\approx -1.96 \text{ kPa}$ ) soil is moist, water is easily accessible. A value of  $-200 \text{ cmH}_2\text{O}$  ( $\approx -19.6 \text{ kPa}$ ), soil is dry, water is harder to extract, plants may experience stress.

Based on Fig. 3, the highest suction potential value occurs at  $X = 10 \text{ cm}$ , while the lowest is found at  $X = 90 \text{ cm}$ . This implies that regions closer to the channel surface exhibit higher suction potential values than those farther away. This indicates that the soil near the channel contains more water than the soil located farther from the channel.

The plots at  $X = 10 \text{ cm}$  and  $X = 30 \text{ cm}$  for channel A are positioned closer to the surface compared to those of channel E. The suction potential values at  $X = 10 \text{ cm}$  and  $X = 30 \text{ cm}$  decrease as the depth of the channel increases, reaching their convergence points. Meanwhile, at  $X = 50 \text{ cm}$ ,  $X = 70 \text{ cm}$ , and  $X = 90 \text{ cm}$ , the suction potential increases toward a convergence point. The decreasing trend of suction potential values shows that less water is absorbed with increasing channel depth.

The suction potential reaches its highest value at  $X = 10 \text{ cm}$  and the lowest value at  $X = 90 \text{ cm}$ . Therefore, two conditions are selected to compare the suction potential values across all channel depths. The Fig. 4 presents a graph of suction potential values at each irrigation channel depth along  $X = 10 \text{ cm}$ .

Fig. 4(a) shows the suction potential values, at  $X = 10 \text{ cm}$ , directly beneath the channel. Channel A has the minimum suction potential value, while channel E exhibits the maximum suction potential. Deeper channels generate higher flux compared to shallower channels. The suction potential value of channel B in layer 1 is not significantly different from that of channel C.

The suction potential value at  $X = 90 \text{ cm}$  reaches the lowest point at Fig. 4(b). It has previously been assumed that there is no other water absorption outside the channel surface, thus the suction potential value at  $X = 90 \text{ cm}$  is lower. Fig. 4(b) shows that the highest to lowest suction potential values along  $X = 90 \text{ cm}$  do not differ significantly from those along  $X = 10 \text{ cm}$ .

Table IV-VIII presents the suction potential values measured at irrigation channels along  $X = 10 \text{ cm}$ , which is closer to the channel surface, and  $X = 90 \text{ cm}$ , which is farther from the channel surface.

The first analysis focuses on channels A and B. In areas near the channel, suction potential decreases with increasing depth. The decrease in layer 1 (SL) is greater than in layer 2 (TSL). Based on Fig. 4, the jump in suction potential values is observed. This jump is considered a transition between two heterogeneous soil layers, occurring around  $Z \approx 150 \text{ cm}$ . This phenomenon is influenced by differences in soil texture, as indicated in Table IV-VIII. At this jump, a significant decrease in suction potential occurs from layer 1 to layer 2. This indicates that layer 2 has a greater water absorption capacity compared to layer 1. The coarser the soil layer, the lower the

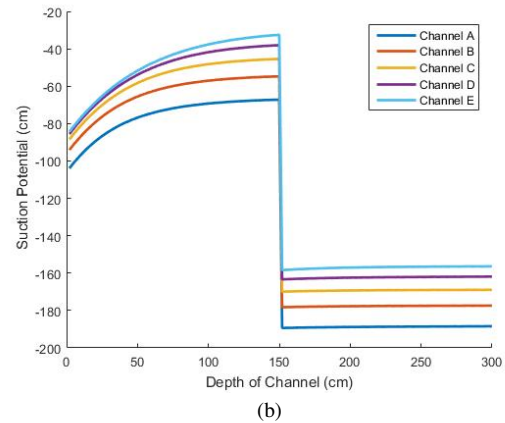
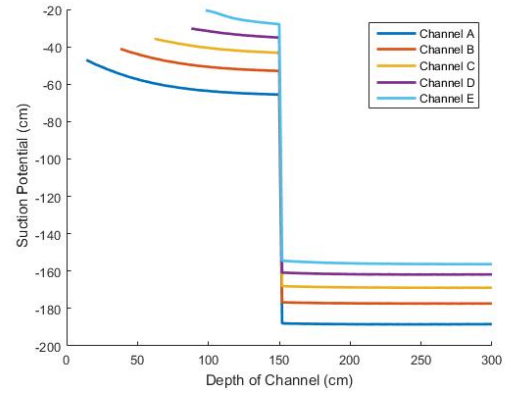


Fig. 4. Suction potential of five types irrigation channels (a)  $X = 10$  and (b)  $X = 90 \text{ cm}$

TABLE IV  
SUCTION POTENTIAL AT SEVERAL POINTS IN CHANNEL A

| Soil | (X,Z)    | $\psi$ (cm) | $\psi$ Change (%) |
|------|----------|-------------|-------------------|
| SL   | (10,40)  | -55.09717   | -17.43%           |
|      | (10,120) | -64.70275   |                   |
|      | (90,40)  | -80.01810   | +14.85%           |
|      | (90,120) | -68.13394   |                   |
| TSL  | (10,200) | -188.40428  | -0.01%            |
|      | (10,280) | -188.42944  |                   |
|      | (90,200) | -188.83664  | +0.17%            |
|      | (90,280) | -188.50620  |                   |

suction potential value. The table below presents the suction potential values for each channel depth variation.

TABLE V  
SUCTION POTENTIAL AT SEVERAL POINTS IN CHANNEL B

| Soil | (X,Z) (cm,cm) | $\psi$ (cm) | $\psi$ Change (%) |
|------|---------------|-------------|-------------------|
| SL   | (10,40)       | -41.59758   | -24.83%           |
|      | (10,120)      | -51.92729   |                   |
|      | (90,40)       | -69.06711   | +19.23%           |
|      | (90,120)      | -55.78230   |                   |
| TSL  | (10,200)      | -177.22480  | -0.07%            |
|      | (10,280)      | -177.35678  |                   |
|      | (90,200)      | -177.70712  | +0.15%            |
|      | (90,280)      | -177.43694  |                   |

The second analysis, conducted on channel C, shows both increases and decreases in suction potential near the channel.

TABLE VI  
SUCTION POTENTIAL AT SEVERAL POINTS IN CHANNEL C

| Soil | (X,Z) (cm,cm) | $\psi$ (cm) | $\psi$ Change (%) |
|------|---------------|-------------|-------------------|
| SL   | (10,62)       | -35.71708   | -17.62%           |
|      | (10,120)      | -42.01157   |                   |
|      | (90,40)       | -62.24924   | +25.02%           |
|      | (90,120)      | -46.67385   |                   |
| TSL  | (10,200)      | -168.70130  | -0.12%            |
|      | (10,280)      | -168.89811  |                   |
|      | (90,200)      | -169.28064  | +0.17%            |
|      | (90,280)      | -168.98635  |                   |

TABLE VII  
SUCTION POTENTIAL AT SEVERAL POINTS IN CHANNEL D

| Soil | (X,Z) (cm,cm) | $\psi$ (cm) | $\psi$ Change (%) |
|------|---------------|-------------|-------------------|
| SL   | (10,88)       | -30.29249   | -10.47%           |
|      | (10,120)      | -33.46320   |                   |
|      | (90,40)       | -58.33421   | +31.84%           |
|      | (90,120)      | -39.75894   |                   |
| TSL  | (10,200)      | -161.64510  | -0.11%            |
|      | (10,280)      | -161.82215  |                   |
|      | (90,200)      | -162.42807  | +0.30%            |
|      | (90,280)      | -161.93897  |                   |

The first condition—a significant increase—is observed where water is still on the channel surface. The second condition—a decrease—is due to water being absorbed by the soil. These conditions are also observed in channels D and E. In all three, the decrease in suction potential in layer 1 (SL) is greater than in layer 2 (TSL).

The third analysis considers all channels in areas farther from the channel. These areas experience an increase in suction potential with increasing depth. This increase in layer 2 becomes smaller and approaches a convergence point. The fourth analysis identifies that channel E has the highest suction potential, while channel A has the lowest. Deeper channels produce higher flux compared to shallower ones.

The overall results indicate that near the channel surface, suction potential decreases as depth increases. This decrease is more pronounced in layer 1 than in layer 2. Conversely, in regions farther from the channel, suction potential increases with depth, with a more substantial increase in layer 1. Changes in layer 2 gradually diminish, approaching a convergence point where suction potential no longer changes significantly.

Thus, based on Fig. 3, 4, and Table IV-VIII, the ranking of suction potential values from highest to lowest is: channel E, channel D, channel C, channel B, and channel A. The higher the suction potential, the faster water is absorbed, and the greater the water content in the soil.

## V. CONCLUSIONS

The water infiltration model in furrow irrigation channels with two layers of heterogeneous soil can be solved using the DRM (Dual Reciprocity Method). The results obtained from the infiltration model are in the form of suction potential values at each soil layer in five types of irrigation channels with varying depths. These values show differences in water absorption caused by soil heterogeneity. Soil layers with finer

TABLE VIII  
SUCTION POTENTIAL AT SEVERAL POINTS IN CHANNEL E

| Soil | (X,Z) (cm,cm) | $\psi$ (cm) | $\psi$ Change (%) |
|------|---------------|-------------|-------------------|
| SL   | (10,98)       | -20.53202   | -22.23%           |
|      | (10,120)      | -25.09587   |                   |
|      | (90,40)       | -56.46858   | +38.16%           |
|      | (90,120)      | -34.92060   |                   |
| TSL  | (10,200)      | -155.82987  | -0.28%            |
|      | (10,280)      | -156.27331  |                   |
|      | (90,200)      | -157.03630  | +0.37%            |
|      | (90,280)      | -156.44969  |                   |

textures tend to have higher suction potential, whereas layers with coarser textures have lower suction potential.

The depth of the irrigation channel affects infiltration in heterogeneous soils. The suction potential values in the two-layer heterogeneous soil infiltration problem, from highest to lowest, are found in channel E, channel D, channel C, channel B, and channel A, with depths of 22.5 cm, 17.5 cm, 12.5 cm, 7.5 cm, and 2.5 cm, respectively. channels with greater depth exhibit higher suction potential values, allowing more water to be absorbed, while shallower channels have lower suction potential. A sudden change in suction potential occurs due to differences in soil texture, represented by varying soil proportions and hydraulic conductivity parameters. More water is absorbed in the first soil layer compared to the second layer.

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