Cubic Spline Model for Kinematic Analysis of the Badminton Smash Movement*

Rahma Dhiyaa Sausan[1], Said Munzir[2], Vera Halfiani[2], and Muhammad Ikhwan[2]

***Abstract*—Smash is one of the most complex movements in badminton. Along with its development, quantitative analysis is needed to obtain the best smash movement. This research aims to analyze the smash movement kinematically through mathematical means. The data in this study are primarily obtained through recording and digitizing data on the smash movement of one Aceh Province badminton athlete. The method used to analyze the data is the cubic spline method. This research produces smash movement kinematic data and analysis of translational and angular curves obtained using a cubic spline interpolation approach for three-dimensional coordinates. The kinematic aspects consisted of body segment position, body segment velocity and acceleration, also angular velocity and acceleration. The conclusion of this research is analysis of the best movement pattern to perform smash in badminton athletes can be conducted by analyzing the translational and angular curves. Those curves obtained by using the cubic spline interpolation approach for position coordinates in three-dimensional space.**

***Index Terms*—Smash, Kinematic, Cubic Spline, Translation, Angular Vellocity and Acceleration.**

I. INTRODUCTION

BADMINTON is one of the most popular sports in the world that requires speed, agility, and dexterity. The sport is played in singles or doubles using a net, racket, and shuttlecock with a variety of hitting techniques [1]–[3]. To achieve the best performance, players must have technical skills, body strength, and quick movements [4]–[6]. One of the most complex techniques in badminton that generates considerable energy and speed is the forehand smash, which is a powerful, swooping blow towards the opponent to end a rally quickly with the aim to kill the opponent [7]–[10]. Rusdiana [11] researched that forehand smash have greater values than backhand smash. Meanwhile Poole [12] analyzed that the smash is the power of a player who can collect numbers for athlete in the match.

The forehand smash, as one of the most complex techniques, involves three phases: acceleration, point of contact or shuttlecock return, and follow-through [5], [13], [14]. Body alignment is required to optimize this stroke, as it involves body segment movements, arm position, feet, and racket usage [8], [15]. Good body positioning during a smash includes standing sideways, bending the arm holding the racket, and alternating foot movements after the jump, with the aim of hitting the shuttlecock as high as possible and aiming sharply at the opponent [16]. The jumping smash is a series of coordination of movements of body parts as a whole that is continuous. Coordination of motion is influenced by skeletal muscles whose function is caused by stimuli carried out by somatic motor nerves in an effort to move the limbs [17].

A forehand smash requires proper wrist coordination to produce a shot that is difficult for the opponent to anticipate. Lack of understanding of the correct technique can lead to injuries such as ACL (Anterior Cruciate Ligagement) and ankle injuries [7], [18]. To prevent those, biomechanics has a strong role and contribution, especially in the field of mathematics by first understanding the movement pattern before smashing. Therefore, it is important to calculate the translation, angular, velocity, and acceleration of body segments while moving. This calculation is known as kinematic, which is one of the branches of science that can formulate optimal movements, such as smash movements in badminton.

Kinematic calculations can be solved through mathematical approaches. To comprehend the kinematic aspects of smash movements, simulation of body segment changes movements can be done through polynomial interpolation. In polynomial functions, the order of the polynomial in interpolation greatly affects the curve results and errors obtained. The higher the order of a polynomial, the smoother the resulting curve will be, so that the error obtained will be smaller and closer to the exact value [16], [19]–[21]. One of the polynomial interpolation methods is the cubic spline method.

The cubic spline method will be used in this study to model the kinematic data of mash movement in three-dimensional coordinates. Cubic spline is preferable because it can produce curves with the expected level of smoothness and can approximate each data better using piecewise polynomial functions [20], [22]–[24]. In addition, the use of a cubic spline will be better used because it has a continuous first and second derivative. In this study, cubic spline functions will be formulated for each body position in three-dimensional coordinates.

In their research, Jeriko and Bima analyzed the biomechanical movements of Barongsai and prayer movements using a cubic spline interpolation approach in two-dimensional space [25], [26]. This research provides an update by using a cubic spline interpolation approach in three-dimensional space. Since this study uses three-dimensional positional coordinates, the coordinates system consists of mutually perpendicular cartesian coordinates, as well as two angles, that is the polar angle (θ) and the azimuth angle (ϕ). In the field of kinematic,

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the x-axis is called the frontal axis, the y-axis is called the sagittal axis, and the z-axis is known as the longitudinal axis. Each of these axes produces translational data, which is the primary data in this study.

Through the primary data obtained, the translational and angular velocities and accelerations of body segments such as shoulders, elbows, wrists, waist, knees, and ankles can be calculated through the cubic spline interpolation function [8], [27]. The results of this study have been verified by comparing the output of the original data curve with the curve resulting from cubic spline interpolation using MATLAB software to ensure the accuracy of the cubic spline approach. This study may provide an insight in the performance improvement analysis of badminton players.

II. METHODOLOGY

The method in this study is divided into two stages, namely the primary data collection stage and forming process of cubic spline algorithm. To obtain the smash movements patten curves in three dimensional coordinates, the collected data will be input into the cubic spline program. The details for each stage of the research are as follows.

A. Data Collection Procedure

The method of taking data on smash movements in badminton uses quantitative analysis of sports biomechanics using manual digitization. Manual digitization is done by giving marks that can be placed on important joints or body parts that you want to study the movement. Before manual digitization is performed, the smash movement process will be recorded by positioning two tripods each from the side direction (sagittal axis) and the rear diagonal direction (frontal axis) of the object with the same distance and height to the object (Fig.1). The 1-second video recording was slowed down by 0.25 seconds, so that a video with a time span of 5 seconds was obtained. The footage was then inputted into the Video For Photos application with a time interval of 0.1 seconds, resulting in 51 frames.

The resulting 51 frames were then fitted with vertical and horizontal rulers on each image from the xz plane (sagittal plane) and the yz plane (frontal plane). This was done to obtain manual digitization of the smash motion recordings. The manual digitization process can be done by marking the observed body segment and then drawing a horizontal line on the image taken from the x-axis and the y-axis simultaneously. Data forming is carried out by making photo visualization on the x-axis and z-axis and on the y-axis and z-axis, so that translational kinematic data of smash movements are obtained.

The translational data obtained will be used to obtain rotational data. Using angular relationships and three-dimensional coordinates, [28]

$$d = |P_1P_2|^2,$$

$$d = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2. \quad (1)$$

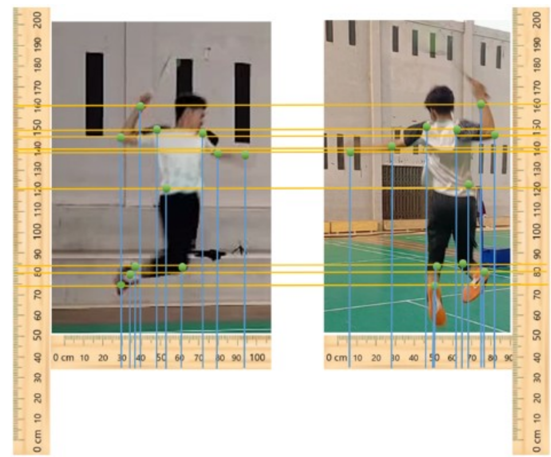


Fig. 1: Digitization manual process of smash movement

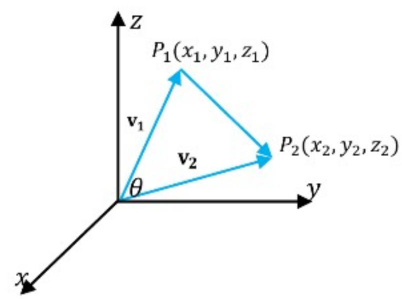


Fig. 2: Relationship between angular and three-dimensional coordinates

Equation (1) is related to position and displacement (vectors). Suppose $v_1 = (x_1, y_1, z_1)$ and $v_2 = (x_2, y_2, z_2)$ are not zero vectors in three-dimensional space, then θ is an angle enclosed by two vectors, namely the vectors v_1 and v_2. Mathematically, this relationship is expressed in the cosine law:

$$|P_1P_2|^2 = (|v_1|)^2 + (|v_2|)^2 - 2|v_1||v_2|cos\theta. \quad (2)$$

To express angles in three-dimensional space, this study uses spherical coordinates where the position of a point in three-dimensional space is expressed using distance $\rho(rho)$, angle $\theta(theta)$, and angle $\phi(phi)$. According to Fig.2, distance (ρ) can be expressed in the equation $\rho = |OP|$, which is the distance from the origin to some other point on the cartesian plane, called P. The angle ρ is the measure of the polar angle of the projection of P on the polar plane, while the angle ϕ is the angle between the positive z-axis and the line segment OP.

The point in the Fig. 3 has a representation in spherical coordinates so it can be written as $P(\rho, \theta, \phi)$. Based on the figure, we can know that $x = |OQ|cos\theta$; $y = |OQ|sin\theta$; $z = |QP|$. Since $|OQ| = \rho sin\phi$ and $|QP| = \rho cos\phi$, the relationship between the spherical coordinates $P(\rho, \theta, \phi)$ and the cartesian coordinates $P(x, y, z)$ can be obtained as follows:

$$x = \rho sin\phi cosX\theta,$$

$$y = \rho sin\phi sin\theta,$$

$$z = \rho cos\phi. \tag{3}$$

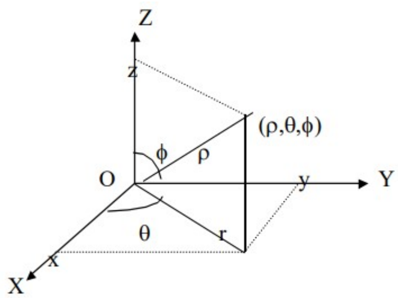


Fig. 3: Relationship between spherical and cartesian coordinate

Transformation of the cartesian coordinates $P(x,y,z)$ into the spherical coordinates $P(\rho,\theta,\phi)$ will obtain the following relationship:

$$\rho = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2},$$

$$tan\theta = \frac{(y_2-y_1)}{(x_2-x_1)},$$

$$cos\phi = \frac{(z_2-z_1)}{\rho},$$

where

$$\rho \geq 0, 0 \leq \theta < 2\pi, 0 \leq \phi \leq \pi. \tag{4}$$

For example, here is the translation data between the right shoulder and right elbow body segments.

TABLE I: Right shoulder-elbow body segment translation

Body segments	*x*	*y*	*z*
Right shoulder	30	35	108
Right elbow	39	28	105

The translational data in Table (I) will be converted into angular data by substituting the data into equation (4), so that the polar angles for the right shoulder and right elbow body segments are obtained.

$$\theta = \tan^{-1}\left(\frac{28-35}{39-30}\right) = -0.6610$$

While the azimuthal angle for the body segment of the right shoulder and right elbow is

$$\phi = \cos^{-1}\left(\frac{105-108}{11.7898}\right) = 1.8281$$

B. The Formation Process of Cubic Spline Algorithm

After having translational and rotational kinematic data, a curve will be formed consisting of cubic spline interpolation. The cubic spline algorithm is divided into two steps. First, creating a program that can run cubic spline calculations in general. Second, using the cubic spline program that has been built to run the cubic spline program with input for smash motion digitization data.

The general algorithm of the cubic spline program is to calculate the cubic spline coefficients so that an interpolation function can be formed. Using the program for the smash motion digitization data, the position function of the body segment will be calculated by applying the first derivative and second derivative for the velocity and acceleration functions. Kinematic data as the dependent variable and time data as the independent variable will be substituted into the cubic spline interpolation obtained previously. The cubic spline interpolation is continuously connected and has the same first and second order derivatives at the vertex points s_i for $i = 1 : n-1$ [29]. The velocity function is obtained through the first derivative of the cubic spline function and the acceleration function is obtained through the second derivative of the cubic spline function.

Mathematically, the general form of a cubic spline can be written as follows:

$$S_i(t) = f_i + b_i(t-t_i) + c_i(t-t_i)^2 + d_i(t-t_i)^3. \tag{5}$$

The initial condition that must be met from the cubic spline is that the spline function must pass through each data point $(t = t_i)$, so that

$$f_i = a_i + b_i(t_i-t_i) + c_i(t_i-t_i)^2 + d_i(t_i-t_i)^3,$$

$$f_i = a_i,$$

The values of adjacent cubic functions must be the same at their vertices. This condition can be modeled for the vertex $i+1$ as follows:

$$f_i + b_ih_i + c_ih_i^2 + d_ih_i^3 = f_{i+1},$$

$$h_i = t_{i+1} - t_i. \tag{6}$$

By applying the first derivative to time t of the body segment cubic spline function equation $S_i(t)$ and satisfying the conditions in equation (6), the body segment velocity spline function is obtained

$$S_i'(t) = b_i + 2c_i(t-t_i) + 3d_i(t-t_i)^2,$$

$$b_i + 2c_i(t-t_i) + 3d_i(t-t_i)^2 = b_{i+1}. \tag{7}$$

By applying the second derivative to time (t) of the body segment cubic spline function equation $S_i(t)$ and satisfying the conditions in equation (6), the segment velocity spline function is obtained

$$S_i''(t) = 2c_i + 6d_i(t-t_i). \tag{8}$$

$$c_i + 3d_ih_i = c_{i+1}, \tag{9}$$

$$d_i = \frac{(c_{i+1}-c_i)}{3h_i}. \tag{10}$$

Substitute equation (10) into (6) to obtain

$$b_i = \frac{f_{i+1}-f_i}{h_i} - \frac{h_i}{3}(2c_i + c_{i+1}). \tag{11}$$

$$b_{i-1} = \frac{f_i-f_{i-1}}{h_{i-1}} - \frac{h_{i-1}}{3}(2c_{i-1} + c_i). \tag{12}$$

Substitute equation (9) into (6) to obtain

$$b_i + (c_{i+1} + c_i)h_i = b_{i+1}.$$
$$b_i = b_{i-1} + (c_i + c_{i-1})h_{i-1}. \tag{13}$$

Substitute equations (11) and (12) into equation (13)

$$3\left(\frac{f_{i+1} - f_i}{h_i}\right) - 3\left(\frac{f_i - f_{i-1}}{h_{i-1}}\right) =$$
$$2h_i c_i + h_i c_{i+1} + 2h_{i-1} c_i + h_{i-1} c_{i-1}. \tag{14}$$

Numerically, equation (14) is a finite difference equation, so it can be defined as follows:

$$f[t_i, t_j] = \frac{(f_i - f_j)}{(t_i - t_j)}.$$

Equation (14) can be written

$$h_i c_{i+1} + 2(h_i + h_{i-1})c_i + h_{i-1} c_{i-1} = 3(f[t_{i+1}, t_i] - f[t_i, t_{i-1}]). \tag{15}$$

where $i = 2, 3, ..., n-2$ and produces $n-3$ a tridiagonal equation with $n-1$ unknown coefficients, $c_1, c_2, \ldots, c_{n-1}$. To get the coefficient c, this approach requires the condition that the second derivative of the first node and the last node are equal to zero. Thus, it can be written as

$$c_1 = c_n = 0.$$

Equation (15) is a system of equations that is linear with respect to c_i, so it can be written in the following matrix form $AX = B$ [30] :

$$\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & 0 \\ h_1 & 2(h_1+h_2) & h_2 & \cdots & 0 & 0 \\ 0 & h_2 & 2(h_2+h_3) & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & \ddots & h_{n-2} & 0 \\ 0 & \cdots & 0 & h_{n-2} & 2(h_{n-2}+h_{n-1}) & h_{n-1} \\ 0 & 0 & \cdots & 0 & 0 & 1 \end{bmatrix}$$
$$\begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_{n-1} \\ c_n \end{bmatrix} = \begin{bmatrix} 0 \\ 3(f[t_3 - t_2] - f[t_2 - t_1]) \\ \vdots \\ 3(f[t_n - t_{n-1}] - f[t_{n-1} - t_{n-2}]) \\ 0 \end{bmatrix} \tag{16}$$

III. RESULTS

In three-dimensional space, the kinematic analyzed include positions with respect to the sagittal, frontal, and longitudinal axes, as well as θ and ϕ. Each velocity curve for each axis is specified with a subscript, i.e., velocity and acceleration of body segments on the frontal axis (v_x and a_x), velocity and acceleration of body segments on the sagittal axis (v_y and a_y), and velocity and acceleration of body segments on the longitudinal axis (v_z and a_z). In addition, cubic spline curves for angles can be obtained from the angles formed during movement between body segments. Of the six observed body segment points, there are five segment points that form angles, both at θ and ϕ. Angular velocity curves were also generated with subscripts, including polar angular velocity and acceleration (ω_θ and α_θ), and azimuthal angular velocity and acceleration (ω_ϕ and α_ϕ).

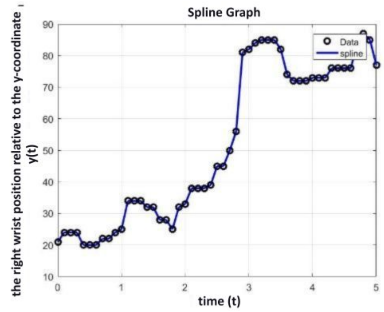


Fig. 4: Right wrist position curve with respect to the sagittal axis

Based on Fig.4, the right wrist at the coordinates-y experiences radial deviation at $2.1s$ to $3.3s$. Radial deviation is a counterclockwise movement of the wrist joint in the sagittal axis with a position towards the backward bending part of the right wrist. This movement occurs in the contact point phase as the wrist position continues to move to the front of the net when viewed from the sagittal axis. The biomechanics of this radial deviation results in a considerable displacement of the right wrist body segment, resulting in a considerable change in position or velocity. When imagining the displacement that occurs in the position of the wrist, one will tend to imagine that the displacement only occurs towards the back to provide force on the arm so that when performing the contact point phase, it will provide the best movement with strong enough power.

In reality, the position of the right wrist viewed from the sagittal axis shows that the right wrist is actually displaced forward and backward during the acceleration, contact point, and follow-up phases, respectively. This caused the curve to change position continuously. In fact, the position of the wrist that was originally in position $21cm$ at 0 s moved to position $77cm$ at $5s$. This indicates that the position of the hand, which was originally at the back and made back and forth movements during the smash movement, will be in the final position at the front of the net (changing the position of the body from the right body at the back of the net to the front of the net).

The velocity curve as shown in Fig. 5 represents the amount of position change for each time interval. The velocity curve obtained through the first derivative of the cubic spline shows a smooth curve due to the quadratic equation obtained through the previous cubic spline approach. In the figure, the curve increases and decreases, which means that there are increasing and decreasing changes in position. The velocity value is negative if the change in position has decreased and positive if the change in position has increased. A zero velocity indicates that there is no change in position or displacement that occurs in that time interval. Based on Fig. 5.a., the maximum velocity on the frontal axis occurs in the left wrist body segment at the fifth second, which is $152.6709cm/s$. This indicates that the left wrist has balanced the right wrist during the follow-up

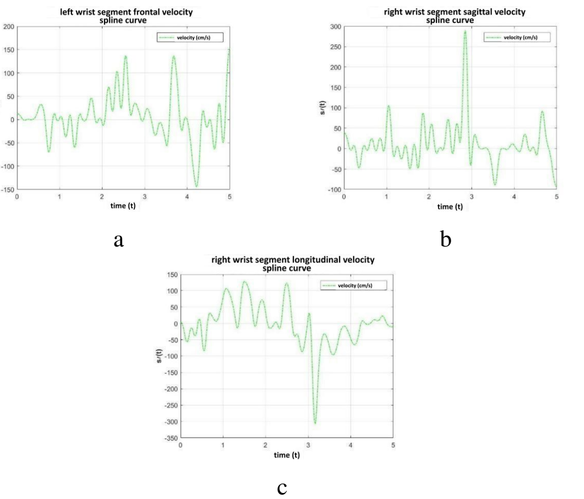


Fig. 5: Velocity curves a) left wrist to frontal axis b) right wrist to sagittal axis c) right wrist to longitudinal axis

phase by making the largest position change from the top to the original position.

In Fig. 5.b., it can be seen that the maximum speed occurs at $2.8s$ of $186.4233cm/s$. This maximum speed occurs due to a considerable change in position from $2.8s$ to $2.9s$. This maximum speed is followed up by a decrease in speed at $2.9s$ to $3s$ with a speed of $-23.1097cm/s$. Through the velocity curve, it can also be seen that the final velocity obtained is $-93.8651cm/s$. This negative value occurs due to a decrease in speed or a position shift towards the back of the net that occurs at the last interval during the follow-up phase. The negative velocity is the minimum velocity generated by the change in position of the right wrist body segment with respect to the sagittal axis.

Fig. 5.c. shows the condition of the largest position change that occurred at $3.2s$ with a minimum value of $-251.64cm/s$, which indicates that the position of the right wrist experienced a significant decrease in position change. Previously, at $3s$, the position change occurred by $20.4004cm/s$, then in the next interval the position change occurred by $-159.6901cm/s$.

An acceleration curve represents the change in velocity that occurs for a given time interval. Just like the velocity curve, the acceleration curve can also be negative and positive. Negative acceleration represents deceleration, which is a change in speed that decreases or slows down (deceleration). Where as a positive acceleration represents a change in velocity that increases with time [31]. Acceleration can also be zero if there is no change in velocity during the time interval. From the Fig.6.a., the change in speed of the right shoulder with respect to the frontal axis amounted to $4905.1182cm/s$ (in seconds). This condition occurs because the follow-up phase given after the smash movement shows the position of the right shoulder moving towards the right front of the body facing the net.

The maximum acceleration in the kinematic of badminton smash motion with respect to the sagittal axis was produced by the right wrist body segment at $2.8s$ (Fig.6.b.). This velocity change occurs at $186.4233cm$ every unit second. In the next interval, the acceleration decelerates which in the sagittal axis

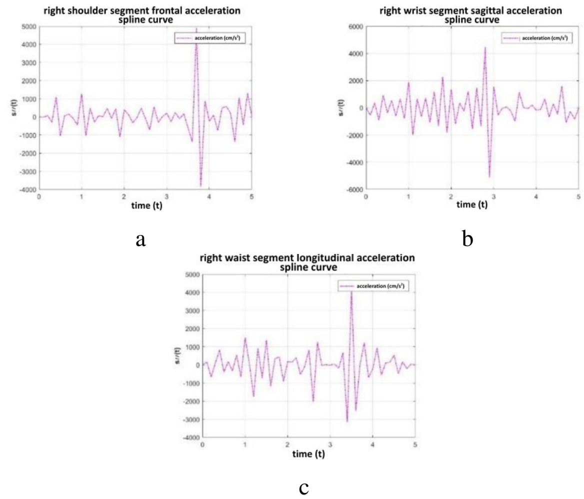


Fig. 6: Acceleration curves of a) right shoulder against frontal axis b) right wrist against sagittal axis c) waist against longitudinal axis

is expressed as a positional displacement towards the back of the net). Based on Fig 6.c., the maximum longitudinal axis acceleration of $4122.4100cm/s^2$ occurred in the waist body segment at the $3.5s$ mark.

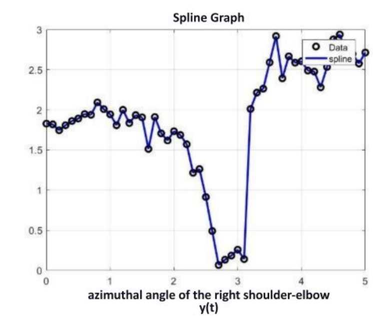
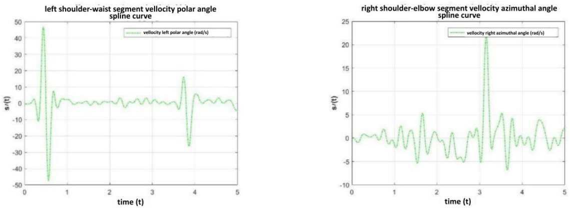


Fig. 7: Body segment azimuthal angle position curve between the right shoulder and right elbow

The angular position curves of the right shoulder and right elbow segments increase and decrease according to Fig.7. This shows that the movement of the right shoulder and right elbow segment segments with respect to the planexy with the longitudinal axis produces movement in these segments. Based on Fig. 4.8.a., the angular position produces the largest angle when $t = 3.1s$ to $t = 3.6s$. The angular velocity refers to the angular displacement with respect to time. Angular acceleration refers to the change in angular velocity with respect to time.

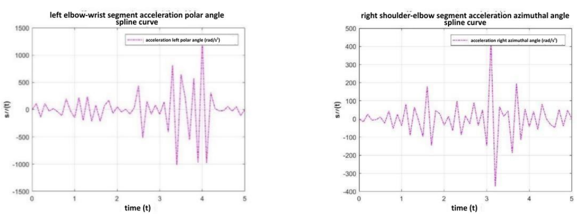
In both figures above, there are positive and negative angular velocities. A positive angular velocity indicates that the segment between two adjacent body segments is moving counterclockwise. Conversely, a negative angular velocity indicates that the movement between two adjacent body segments is clockwise. An angular velocity of zero means that there is no displacement of the angular position between the body segments that form an angle.



a b

Fig. 8: Angular velocity curves a) polar between shoulder and left waist body segments b) azimuthal between shoulder and right elbow body segments

The maximum θ velocity occurs from $0.4s$ to $0.5s$ which is $27.8800 rad/s$. Thus, the change in position occurs counterclockwise (Fig.8.a.). The largest displacement ϕ occurs from $3.1s$ to $3.2s$ with a positive value which means that the rate of angular displacement occurs counterclockwise. Therefore, the maximum angular velocity also occurs at that second of $13.3836 rad/s$ with a maximum velocity change of $428.5853 rad/s^2$ at $3.1s$ (Fig.8.b.).



a b

Fig. 9: Angular acceleration curves a) polar between elbow and left wrist body segments b) azimuthal between shoulder and right elbow body segments

In the figure above, there are positive and negative angular accelerations. A positive angular acceleration indicates that the change in angular velocity between the segments of the left elbow and left wrist or right shoulder and right elbow increases proportionally. Conversely, a negative angular acceleration indicates that the change in angular velocity of the segment between the left elbow and the left wrist or the right shoulder and the right elbow decreases proportionally. Similar to positional acceleration, angular acceleration of zero means that the right shoulder and right elbow segments at each angle do not experience a change in angular velocity (the angular velocity is fixed). In this case, the angular acceleration is zero at $0s$ and $5s$ (before and after the movement is performed, there is no change in angular velocity).

In Fig. 9.a., the maximum θ acceleration of $1286.4801 rad/s^2$ occurs between the elbow and left wrist body segments at the fourth second. While the maximum ϕ acceleration in Fig.9.b. occurs between the shoulder and right elbow body segments $428.5853 rad/s^2$ at the $3.1s$. Based on the analysis of both angles in three-dimensional space, the development of greater angular velocity and acceleration occurred at θ. This is due to the side angle (θ) between the body segments being more dominant when compared to the upright angle generated at the angle formed between the plane-xy and the longitudinal axis (ϕ).

IV. CONCLUSIONS

The analysis of badminton smash movement using cubic spline interpolation provides smooth and accurate kinematic curves, enabling detailed examination of velocity and acceleration patterns. The general pattern reveals that the right wrist plays a dominant role by exhibiting the highest velocity and acceleration, particularly along the sagittal axis, which is critical for generating powerful smashes. Meanwhile, the left wrist shows notable velocity in the frontal axis, contributing to balance and control during the movement. Proximal body segments such as the right shoulder and waist also demonstrate significant acceleration, highlighting their essential role in initiating and supporting the smash through rotational and force transfer mechanisms.

The use of cubic spline interpolation as a mathematical tool offers an improved representation of movement dynamics compared to traditional manual digitization methods. However, this study is limited by its reliance on a single athlete and manual digitalization, which restricts the generalizability of the findings. Future research should aim to include a larger and more diverse sample of athletes and incorporate modern motion capture systems for more precise and automated data collection. Validating these findings with additional biomechanical measurement tools such as IMU sensors or 3D cameras could further enhance the understanding of optimal smash mechanics and support the development of more effective training protocols. On the other hand, this research can be used as comparative material for further studies in this field to be analyzed further.

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