A Comparative Study of Fuzzy Chain Ladder and Fuzzy Bornhuetter-Ferguson Methods for Claims Reserving

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ABSTRACT – Accurate estimation of claim reserves is critical for the solvency of non-life insurance companies. Classical deterministic methods, such as Chain Ladder (CL) and Bornhuetter-Ferguson (BF), often fail to capture the inherent vagueness in actuarial judgments regarding development patterns and prior information. Fuzzy Set Theory, particularly through Triangular Fuzzy Numbers (TFNs), offers a formal framework to model this imprecision. This study conducts an empirical comparative analysis of two fuzzy reserving methods: the Fuzzy Chain Ladder (FCL) and the Fuzzy Bornhuetter-Ferguson (FBF). The FCL method fuzzifies the development factors, while the FBF method extends fuzzification to both the development pattern and the prior estimate of ultimate losses. Both methods are applied to a real-world liability insurance claims dataset from an Indonesian company, structured into a run-off triangle. The performance is evaluated by comparing the central reserve estimates, the total model uncertainty, and the asymmetry of the fuzzy output intervals. The results indicate that the FCL method produces a more conservative and volatile central reserve, approximately 12.3% higher than the FBF estimate under a risk-neutral assumption. More importantly, the FBF method demonstrates superior stability, with a total uncertainty measure about 14.7% lower than FCL, and exhibits an asymmetric uncertainty structure in which the right spread is consistently narrower, making its defuzzified reserve less sensitive to the actuary's risk attitude. The study concludes that the FCL method is suitable as a transparent, data-driven benchmark, whereas the FBF method is recommended for generating more robust and risk-sensitive forecasts when credible prior information is available.

Keywords– Claims Reserving, Fuzzy Chain Ladder, Fuzzy Bornhuetter-Ferguson, Triangular Fuzzy Number

I. INTRODUCTION

Risk is an unavoidable part of human life and can arise in many forms, such as illness, property damage, or death. When individuals or organizations face such events, their financial stability can be undermined, making effective risk-mitigation mechanisms essential. Insurance functions not only as a financial protection instrument but also as a cooperative system that enables the sharing and redistribution of risk among policyholders and institutions.

In the non-life insurance sector, companies must ensure adequate reserves are set aside to cover future claim payments. Unlike life insurance, where claim settlement tends to occur shortly after the insured event, non-life claims often experience significant delays between occurrence, reporting, and final settlement. These delays create outstanding claim liabilities that impact an insurer's financial statements and solvency indicators. Therefore, determining an appropriate level of technical reserves is a central task for actuaries, who must balance the risks of both underestimation and overestimation.

A wide range of reserving techniques have been developed to support this task. The Chain Ladder (CL) [1], [2] and Bornhuetter-Ferguson (BF) [3] methods are among the most widely applied due to their conceptual simplicity and empirical reliability across various lines of business [4]. The CL method relies solely on historical development patterns, whereas the BF method supplements these empirical patterns with a priori information from expert judgment, market indicators, or similar portfolios. Together, CL and BF provide essential tools for modern actuarial reserving.

However, both approaches require estimates of development factors and ultimate claims, which in practice often involve a degree of vagueness or ambiguity. Actuaries frequently rely on professional judgment, experience, or external information that is not perfectly precise. Classical reserving methods treat these inputs as crisp, deterministic values, even though real-world information is often imprecise. This mismatch highlights the need for approaches that can represent and propagate such uncertainty within the reserving process.

Fuzzy Set Theory (FST), introduced by Zadeh (1965) [5], provides a suitable framework for modeling imprecise or linguistically defined information. Triangular Fuzzy Numbers (TFNs) are particularly useful in this context because they maintain computational tractability while capturing the range and shape of uncertainty in actuarial judgments. A TFN is defined by its mode (central value), left spread, and right spread. This representation provides a natural way to augment classical point estimates with a formal range of uncertainty.

Although the use of fuzzy methods has grown in actuarial research, their application in claims reserving remains relatively limited. Earlier work primarily explored Fuzzy Regression (FR) approaches [6], [7], [8], [9], [10]. A more direct integration of FST into reserving methods was later proposed by Heberle and Thomas (2014, 2016) [11], [12], who extended the CL and BF methods into the Fuzzy Chain Ladder (FCL) and Fuzzy Bornhuetter-Ferguson (FBF) frameworks. These methods allow actuaries to incorporate vagueness not only in development patterns but also in prior estimates of ultimate losses. However, a focused empirical comparison of these two fuzzy methods assessing their relative performance and stability when applied to real world claims data is still underexplored in the literature.

Therefore, this study conducts a comparative empirical study of the FCL and FBF approaches to evaluate their effectiveness in representing and managing uncertainty in non-life claim reserving. The FCL method introduces fuzziness specifically into development factors, while the FBF method extends fuzzification to both development patterns and prior estimates of ultimate losses. By applying both methods to actual claims data, this study seeks to provide practical insights into the conditions under which each fuzzy method offers more robust and reliable reserve estimates, thereby providing empirical evidence and practical guidance in selecting an appropriate fuzzy reserving method amidst imprecision.

II. LITERATURE REVIEW

A. Run-Off Triangle and Classical Reserving Foundation

The fundamental data structure for non-life insurance reserving is the run-off triangle [4], [13]. It organizes historical claims experience by two dimensions: accident period (or origin period) i and development period (delay period) j. Let $D_{i,j}$ denote the incremental claims amount for accidents that occurred in period i and were paid in development period j. The set of all observable data at the current valuation date $t = I$ is $O_I = \{D_{i,j} | i + j \leq I\}$, forming an upper-triangular matrix as shown in Table 1.

Table 1 General Structure of a Run-Off Triangle (Incremental Claims)

Accident Period (i)	Development Period						
	1	**2**	$\cdots$	j	$\cdots$	$J-1$	J
1	D_{11}	D_{12}	$\cdots$	D_{1j}	$\cdots$	$D_{1(J-1)}$	D_{1J}
2	D_{21}	D_{22}	$\cdots$	D_{2j}	$\cdots$	$D_{2(J-1)}$	
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$		
i	D_{i1}	D_{i2}	$\cdots$	D_{ij}			
$\vdots$	$\vdots$	$\vdots$	$\vdots$				
$I-1$	$D_{(I-1)1}$	$D_{(I-1)2}$					
I	D_{I1}						

The cumulative claims up to development period j are defined as $C_{i,j} = \sum_{k=1}^{j} D_{i,k}$. The core challenge of claims reserving is to project the unknown values in the lower triangle to estimate total outstanding liabilities.

B. Fuzzy Set Theory and Triangular Fuzzy Numbers

Fuzzy Set Theory (FST), introduced by Zadeh [5], provides a mathematical framework for quantifying and processing imprecise or vague information. A Triangular Fuzzy Number (TFN) is a specific and computationally efficient type of fuzzy number. Following the notation of Heberle & Thomas [11], a TFN $\tilde{A}$ is defined as:

$$\tilde{A} = (a, l_a, r_a),$$

where a represents the mode (central value), l_a denotes the left spread, and r_a denotes the right spread. The set of all possible values for the fuzzy number is contained within the interval $[a - l_a, a + r_a]$.

C. Fuzzy Chain Ladder (FCL) Method

The classical Chain Ladder (CL) method operates on the run-off triangle of cumulative claims, $C_{i,j}$. It assumes the existence of stable development factors f_j [1], [2]:

$$E[C_{i,j+1} | C_{i,j}] = f_j C_{i,j}. \quad (1)$$

While robust, this method treats the development factors f_j as precise, crisp values, failing to capture the vagueness inherent in actuarial judgment.

To formalize this uncertainty, Heberle & Thomas (2014) [11], introduced the Fuzzy Chain Ladder (FCL) method. The core innovation is to model each development factor as a Triangular Fuzzy Number (TFN):

$$\tilde{f}_j = \left(f_j, l_{f_j}, r_{f_j}\right), \quad (2)$$

where f_j is the mode (central estimate), and l_{f_j}, r_{f_j} are the left and right spreads.

The estimators proposed by Heberle & Thomas retain the classical CL estimator as the mode but augment it with symmetric spreads derived from the observed incremental claims:

$$\hat{f}_j = \frac{\sum_{i=1}^{I-j-1} C_{i,j+1}}{\sum_{i=1}^{I-j-1} C_{i,j}}, \quad (3)$$

$$\hat{l}_{\hat{f}_j} = \hat{r}_{\hat{f}_j} = \frac{\sum_{i=1}^{I-j-1} D_{i,j+1}}{\sum_{i=1}^{I-j-1} C_{i,j}}. \quad (4)$$

Projecting ultimate claims requires fuzzy arithmetic. The fuzzy ultimate cumulative claim for accident period i is given by:

$$\hat{\tilde{C}}_{i,J} = \tilde{C}_{i,I-1} \otimes_{j=I-i}^{J-1} \hat{\tilde{f}}_j. \quad (5)$$

The product of the relevant fuzzy development factors itself forms a TFN:

$$\otimes_{j=I-i}^{J-1} \hat{\tilde{f}}_j = \left(\hat{F}_{I-i}^{J-1}, \hat{l}_{\hat{F}_{I-i}^{J-1}}, \hat{r}_{\hat{F}_{I-i}^{J-1}}\right), \quad (6)$$

where the components are calculated as:

$$\hat{F}_{I-i}^{J-1} = \prod_{j=I-i}^{J-1} \hat{f}_j, \tag{7}$$
$$l_{\hat{F}_{I-i}^{J-1}} = \hat{F}_{I-i}^{J-1} - 1, \tag{8}$$
$$r_{\hat{F}_{I-i}^{J-1}} = \prod_{j=I-i}^{J-1}(2\hat{f}_j - 1) - \hat{F}_{I-i}^{J-1}. \tag{9}$$

The fuzzy outstanding reserve for accident period i is then the fuzzy difference between the projected ultimate and the cumulative claims already paid or reported:

$$\hat{\tilde{R}}_i = \hat{\tilde{C}}_{i,J} \ominus \tilde{C}_{i,I-i}, \tag{10}$$

with $\tilde{C}_{i,I-i} = (C_{i,I-i}, 0,0)$. The total portfolio reserve is the fuzzy sum across all periods:

$$\hat{\tilde{R}} = \oplus_{i=1}^{I} \hat{\tilde{R}}_i. \tag{11}$$

As financial statements require deterministic figures, the fuzzy reserves must be defuzzified. Heberle & Thomas employ an expected value operator E_β, parameterized by a risk attitude coefficient $\beta \in [0,1]$:

$$E_\beta(\tilde{C}_{i,j+1} | \tilde{C}_{i,j}) = C_{i,j}\left(\hat{f}_j - \frac{1-\beta}{2} l_{\hat{f}_j} + \frac{\beta}{2} r_{\hat{f}_j}\right). \tag{12}$$

A significant contribution of their framework is a dedicated measure to quantify the vagueness of the fuzzy output, analogous to a prediction error. For each accident period, the uncertainty is:

$$Unc_K(\tilde{C}_{i,J}|\mathcal{O}_I) = \frac{1}{2} K \tilde{C}_{i,I-i}\left(l_{\hat{F}_{I-i}^{J-1}} + r_{\hat{F}_{I-i}^{J-1}}\right), \tag{13}$$

where $K \in \mathbb{R}^+$ is a scaling parameter. The total uncertainty is the aggregate:

$$Unc_K\left(\sum_{i=1}^{I} \tilde{C}_{i,J} | \mathcal{O}_I\right) = \sum_{i=1}^{I}\left(Unc_K(\tilde{C}_{i,J}|\mathcal{O}_I)\right). \tag{14}$$

D. Fuzzy Bornhuetter-Ferguson (FBF) Method

The classical Bornhuetter-Ferguson (BF) method improves stability by incorporating an a priori estimate of ultimate losses, v_i, alongside the empirical development pattern γ_j [3]. The Fuzzy Bornhuetter-Ferguson (FBF) method, developed by Heberle & Thomas (2016) [12], extends this by fuzzifying both informational components.

In the FBF method, both the prior estimate and the development pattern are represented as TFNs:

$$\tilde{v}_i = (v_i, l_{v_i}, r_{v_i}), \quad \tilde{\gamma}_j = (\gamma_j, l_{\gamma_j}, r_{\gamma_j}). \tag{15}$$

The fuzzy prior estimator, $\hat{\tilde{v}}_i = (\hat{v}_i, l_{\hat{v}_i}, r_{\hat{v}_i})$, often sets its mode $\hat{v}_i$ according to exposure-based methods. A common approach defines it as the product of the number of claims from the CL method N_i, and an estimated mean claim $\bar{C}$ [14]:

$$\hat{v}_i = N_i \times \bar{C}. \tag{16}$$

Following the FBF framework, the spreads $l_{\hat{v}_i}$ and $r_{\hat{v}_i}$ were specified as expert-defined uncertainty margins around the prior estimate. In this study, symmetric spreads were adopted and increased across accident periods to reflect the greater uncertainty associated with less mature claim development.

The fuzzy development pattern is constructed consistently with the FCL framework, derived from the inverse fuzzy development factors:

$$\hat{\tilde{\gamma}}_j = \otimes_{k=j}^{J-1} \hat{\tilde{f}}_k^{\,-1}, \tag{17}$$

for $j = 1,2,\cdots,J-1$ and $\hat{\tilde{\gamma}}_j = (1,0,0)$.

The fuzzy ultimate claim under the FBF methodology is then calculated using fuzzy arithmetic, blending the paid-to-date amount with the fuzzy prior:

$$\hat{\tilde{C}}_{i,J} = \tilde{C}_{i,I-i} \oplus (1 \ominus \hat{\tilde{\gamma}}_{I-i}) \odot \hat{\tilde{v}}_i. \tag{18}$$

The corresponding fuzzy reserve and the total aggregate reserve are obtained analogously to the FCL method:

$$\hat{\tilde{R}}_i = \hat{\tilde{C}}_{i,J} \ominus \tilde{C}_{i,I-i}, \tag{19}$$
$$\hat{\tilde{R}} = \oplus_{i=1}^{I} \hat{\tilde{R}}_i. \tag{20}$$

Defuzzification follows the same E_β operator principle, applied to the FBF structure:

$$E_\beta(\tilde{C}_{i,j+1} | \tilde{C}_{i,j}) = C_{i,j}\left(\hat{\gamma}_j - \frac{1-\beta}{2} l_{\hat{\gamma}_j} + \frac{\beta}{2} r_{\hat{\gamma}_j}\right). \tag{21}$$

Finally, the uncertainty for FBF, which accounts for vagueness in both the prior and the development pattern, is given by:

$$Unc(\tilde{C}_{i,J}|\mathcal{O}_I) = \frac{1}{2}\left[(1-\gamma_{I-i})(l_{\hat{v}_i} + r_{\hat{v}_i}) + (l_{\hat{\gamma}_{I-i}} + r_{\hat{\gamma}_{I-i}})\hat{v}_i + l_{\hat{\gamma}_{I-i}} r_{\hat{v}_i} - r_{\hat{\gamma}_{I-i}} l_{\hat{v}_i}\right]. \tag{22}$$

The total uncertainty is the sum across all accident periods i.

III. METHODOLOGY

A. Data

This study employs a comparative research design to evaluate the performance of the Fuzzy Chain Ladder (FCL) and Fuzzy Bornhuetter-Ferguson (FBF) methods. The analysis is applied to a real-world dataset from the liability insurance portfolio of an Indonesian general insurance company. The raw data includes fields necessary for reserving analysis: claim ID, accident date, reporting date, and a complete history of payment transactions. To transform this individual-level data into the structured run-off triangle format required for the analysis, the data processing and aggregation were performed using the Rapp actuarial package [15]. This process involved aggregating all payment amounts by their corresponding accident period (i) and development period (j) to form the incremental claims matrix, $D_{i,j}$.

B. Analysis Framework

The analytical workflow for this comparative study follows a structured sequence of steps, as visualized in the research flowchart (Figure 1). This ensures a systematic application of both fuzzy methods to the same dataset, enabling a direct and fair comparison.

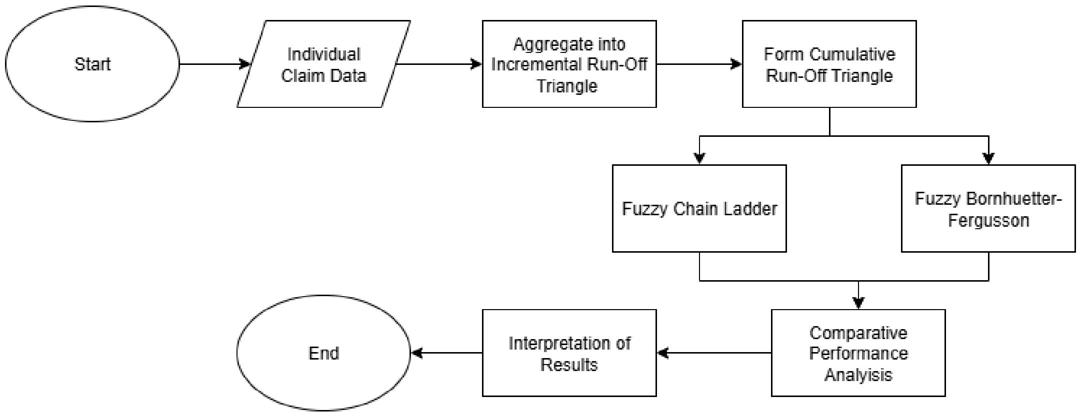

Figure 1 Research Flowchart

The analysis begins with data preparation to form the run-off triangle. Following this, the fuzzy parameters for each method are estimated. For FCL, this involves calculating the fuzzy development factors $\tilde{f}_j$. For FBF, it requires deriving the fuzzy development pattern $\tilde{\gamma}_j$ from the FCL factors and constructing the fuzzy prior ultimate estimate $\tilde{v}_i$.

Subsequently, both methods are used to project fuzzy ultimate claims and calculate the corresponding fuzzy outstanding reserves ($\tilde{R}^{FCL}$ and $\tilde{R}^{FBF}$). These fuzzy outputs are then defuzzified using the expected value operator E_β to obtain crisp reserve values suitable for financial reporting. The inherent uncertainty for each method is quantified using the respective Unc measure.

The final and critical step is the comparative evaluation, where the outputs of FCL and FBF are directly contrasted based on (1) the central reserve estimate, (2) the total uncertainty index (Unc) and (3) the risk profile evident in the symmetry and width of their fuzzy reserve intervals.

IV. RESULTS AND DISCUSSIONS

The liability insurance dataset used in this study was obtained from an Indonesian general insurance company. The individual claim-level records were aggregated into a 6×6 incremental run-off triangle (in millions of Rupiah) using the Rapp [15], as presented in Table 2. The data exhibits a pattern typical of long-tail liability insurance, with substantial payments in the early development periods and considerable variability across accident periods, highlighting the uncertainty inherent in the claim development process.

Table 2 Run-Off Triangle of Liability Insurance Claim Data (in Millions Rupiah)

Accident	Development Period					
Period (i)	**1**	**2**	**3**	**4**	**5**	**6**
1	115.06	421.29	37.68	4.22	0.00	0.00
2	3861.01	1664.80	36.58	7.17	2.16	
3	3021.57	1031.04	13.08	0.00		
4	4091.71	1191.53	1.44			
5	2005.14	850.32				
6	2149.87					

For the implementation of the FBF method, a priori information is required to construct the fuzzy prior ultimate loss estimates. The central values of the prior estimates, v_i , were calculated using Equation (16). Following the framework proposed by Heberle and Thomas [12], these prior estimates are treated as approximate rather than exact quantities. Consequently, fuzziness is introduced into the prior information by representing each estimate as a Triangular Fuzzy Number (TFN), $\tilde{v}_i = (v_i, l_{v_i}, r_{v_i})$.

The left and right spreads represent the uncertainty associated with the prior estimates and reflect the actuary's confidence in the available information. In this study, symmetric spreads were adopted to avoid introducing directional bias into the prior estimates. Larger spreads were assigned to more recent accident periods to reflect the greater uncertainty associated with less mature claim development and the limited amount of observed claim information available for those periods. The resulting fuzzy prior information is presented in Table 3.

Table 3 Given A Priori Information $\tilde{v}_i = (v_i, l_{v_i}, r_{v_i})$

i	v_i	l_{v_i}	r_{v_i}
1	805.60	0	0
2	5295.13	100	100
3	5504.92	200	200
4	4198.85	300	300
5	2799.45	400	400
6	2800.29	500	500

The fuzzy development factors ($\tilde{f}_j$) for the FCL method and the fuzzy development pattern ($\tilde{\gamma}_j$) for the FBF method are presented in Table 4 and Table 5. A direct comparison shows that the estimated fuzzy parameters have different uncertainty profiles. For instance, in development period 1, the FCL factor $\tilde{f}_1$ has a left spread of 0.3940 and a right spread of 0.3940. In contrast, the corresponding FBF development pattern $\tilde{\gamma}_1$ has a left spread of 0.1609 and a right spread of 0.2878. The total spread (sum of left and right) for the FBF parameter is narrower than that of the FCL parameter. This structural difference arises from the mathematical derivation of $\tilde{\gamma}_j$ from $\tilde{f}_j$, indicating that the derived development pattern ($\tilde{\gamma}_j$) has a narrower range of uncertainty than the original development factors ($\tilde{f}_j$).

Table 4 Fuzzy Chain Ladder (FCL) Development Factors

$\tilde{\hat{f}}_j$	Development Period				
	1	2	3	4	5
$\hat{f}_j$	1.3940	1.0058	1.0011	1.0004	1.0000
$\hat{l}_{\hat{f}_j}$	0.3940	0.0058	0.0011	0.0004	0.0000
$\hat{r}_{\hat{f}_j}$	0.3940	0.0058	0.0011	0.0004	0.0000

Table 5 Fuzzy Bornhuetter-Ferguson (FBF) Development Factors

$\tilde{\hat{\gamma}}_j$	Development Period				
	1	2	3	4	5
$\hat{\gamma}_j$	0.7122	0.9928	0.9985	0.9996	1.0000
$\hat{l}_{\hat{\gamma}_j}$	0.1609	0.0071	0.0015	0.0004	0.0000
$\hat{r}_{\hat{\gamma}_j}$	0.2878	0.0072	0.0015	0.0004	0.0000

Based on these fuzzy parameters, the unobserved lower-right cells of the run-off triangle were projected to estimate future cumulative claims. Each prediction $\tilde{C}_{i,j}$ is expressed as a fuzzy number, enabling a realistic reflection of the uncertainty embedded in the claim process. The completed run-off triangles from the FCL and FBF methods are shown in Table 6 and Table 7, respectively.

The tables show a clear difference in how uncertainty grows in the projections. The FCL results (Table 6) show wider fuzzy intervals, meaning a larger range of possible cumulative claim values, especially for the most recent accident periods where there is less historical data to rely on. In contrast, the FBF projections (Table 7) have narrower and more stable intervals. This narrowing stems primarily from the arithmetic structure of the two methods. The FCL projection multiplies a sequence of fuzzy development factors, and each fuzzy product enlarges the spreads cumulatively, so vagueness compounds toward the most recent accident periods. The FBF projection instead relies more on fuzzy addition and on fewer fuzzy multiplications, since the prior estimate anchors the ultimate claim directly; as a result, the spreads accumulate far less, and the projected intervals stay narrower and more stable.

Table 6 Completed Run-Off Triangle Using FCL (Fuzzy Cumulative Claims, in Millions Rupiah)

$\tilde{\hat{C}}_{i,j}$	Development Period					
	1	2	3	4	5	6
$\hat{C}_{1,j}$	115.06	536.35	574.03	578.24	578.24	578.24
$\hat{l}_{\hat{C}_{1,j}}$	0	0	0	0	0	0
$\hat{r}_{\hat{C}_{1,j}}$	0	0	0	0	0	0
$\hat{C}_{2,j}$	3861.01	5525.81	5562.39	5569.56	5571.72	5571.7225
$\hat{l}_{\hat{C}_{2,j}}$	0	0	0	0	0	0.0000
$\hat{r}_{\hat{C}_{2,j}}$	0	0	0	0	0	0.0000
$\hat{C}_{3,j}$	3021.57	4052.60	4065.68	4065.68	4067.1136	4067.1136
$\hat{l}_{\hat{C}_{3,j}}$	0	0	0	0	1.4317	1.4317
$\hat{r}_{\hat{C}_{3,j}}$	0	0	0	0	1.4317	1.4317

$\widehat{\tilde{C}}_{i,j}$	Development Period					
	1	2	3	4	5	6
$\hat{C}_{4,j}$	4091.71	5283.25	5284.69	5290.5868	5292.4498	5292.4498
$\hat{l}_{\hat{C}_{4,j}}$	0	0	0	5.8968	7.7598	7.7598
$\hat{r}_{\hat{C}_{4,j}}$	0	0	0	5.8968	7.7640	7.7640
$\hat{C}_{5,j}$	2005.14	2855.46	2871.9218	2875.1263	2876.1388	2876.1388
$\hat{l}_{\hat{C}_{5,j}}$	0	0	16.4642	19.6687	20.6812	20.6812
$\hat{r}_{\hat{C}_{5,j}}$	0	0	16.4642	19.7055	20.7318	20.7318
$\hat{C}_{6,j}$	2149.87	2996.8738	3014.1534	3017.5167	3018.5792	3018.5792
$\hat{l}_{\hat{C}_{6,j}}$	0	847.0068	864.2863	867.6496	868.7122	868.7122
$\hat{r}_{\hat{C}_{6,j}}$	0	847.0068	874.0537	879.3676	881.0495	881.0495

Table 7 Completed Run-Off Triangle Using FBF (Fuzzy Cumulative Claims, in Millions Rupiah)

$\widehat{\tilde{C}}_{i,j}$	Development Period					
	1	2	3	4	5	6
$\hat{C}_{1,j}$	115.06	536.35	574.03	578.24	578.24	578.24
$\hat{l}_{\hat{C}_{1,j}}$	0	0	0	0	0	0
$\hat{r}_{\hat{C}_{1,j}}$	0	0	0	0	0	0
$\hat{C}_{2,j}$	3861.01	5525.81	5562.39	5569.56	5571.72	5571.7225
$\hat{l}_{\hat{C}_{2,j}}$	0	0	0	0	0	0.0000
$\hat{r}_{\hat{C}_{2,j}}$	0	0	0	0	0	0.0000
$\hat{C}_{3,j}$	3021.57	4052.60	4065.68	4065.68	4067.6197	4067.6197
$\hat{l}_{\hat{C}_{3,j}}$	0	0	0	0	1.9378	1.9378
$\hat{r}_{\hat{C}_{3,j}}$	0	0	0	0	2.0772	2.0772
$\hat{C}_{4,j}$	4091.71	5283.25	5284.69	5289.3683	5290.8463	5290.8463
$\hat{l}_{\hat{C}_{4,j}}$	0	0	0	7.4222	6.1564	6.1564
$\hat{r}_{\hat{C}_{4,j}}$	0	0	0	8.4983	7.0203	7.0203
$\hat{C}_{5,j}$	2005.14	2855.46	2871.4828	2874.6019	2875.5874	2875.5874
$\hat{l}_{\hat{C}_{5,j}}$	0	0	23.0529	20.8330	20.1297	20.1297
$\hat{r}_{\hat{C}_{5,j}}$	0	0	29.7134	26.5943	25.6088	25.6088
$\hat{C}_{6,j}$	2149.87	2935.6234	2951.6534	2954.7735	2955.7592	2955.7592
$\hat{l}_{\hat{C}_{6,j}}$	0	818.6408	808.5237	806.5253	805.8921	805.8921
$\hat{r}_{\hat{C}_{6,j}}$	0	695.0840	679.0540	675.9339	674.9482	674.9482

From these completed triangles, the fuzzy reserves for each accident period were derived. The results, summarized in Table 8, reveal substantive differences between the two methodologies. The FCL method produces a more conservative total central reserve (mode) of IDR 898.5848 million, compared to IDR 834.1160 million from the FBF method, a difference of approximately 7.7%. More notably, the two methods differ in the shape of their uncertainty. The FCL reserves display nearly symmetric spreads (e.g., for accident period 6: $l_{R_6} = 868.7122, r_{R_6} = 881.0495$), so the perceived scope for the reserve to lie above or below its mode is comparable. The FBF reserve for the same period is asymmetric, with a right spread ($r_{R_6} = 674.9482$) markedly narrower than its left spread ($l_{R_6} = 805.8921$). FBF therefore concentrates a smaller share of its vagueness on the upper side of the central estimate, producing a more compact range of plausible values above the mode, whereas FCL distributes its vagueness more evenly. This asymmetry characterizes how each method allocates its vagueness around the central estimate.

Table 8 Predicted Fuzzy Reserves by Accident Period (in Millions Rupiah)

Accident Period (i)	FCL Reserves $\widehat{\tilde{R}}^{FCL}$			FBF Reserves $\widehat{\tilde{R}}_i^{FBF}$		
	$\hat{R}_i$	$\hat{l}_{\hat{R}_i}$	$\hat{r}_{\hat{R}_i}$	$\hat{R}_i$	$\hat{l}_{\hat{R}_i}$	$\hat{r}_{\hat{R}_i}$
1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	1.4317	1.4317	1.4317	1.9378	1.9378	2.0772
4	7.7598	7.7598	7.7640	6.1564	6.1564	7.0203

Accident Period (i)	FCL Reserves $\hat{\hat{R}}^{FCL}$			FBF Reserves $\hat{\hat{R}}_i^{FBF}$		
	$\hat{R}_i$	$\hat{l}_{\hat{R}_i}$	$\hat{r}_{\hat{R}_i}$	$\hat{R}_i$	$\hat{l}_{\hat{R}_i}$	$\hat{r}_{\hat{R}_i}$
5	20.6812	20.6812	20.7318	20.1297	20.1297	25.6088
6	868.7122	868.7122	881.0495	805.8921	805.8921	674.9482
Σ	898.5848	898.5848	910.9769	834.1160	834.1160	709.6545

The sensitivity of the defuzzified total reserve to the actuary's risk attitude (β) and the total uncertainty (Unc) are presented in Table 9. As expected, reserves increase with higher β (greater risk aversion). Consistently, the FCL method yields higher expected reserves across all risk attitudes, aligning with its more conservative behavior in this dataset. For a risk-neutral stance ($\beta = 0.5$), the FCL reserve is IDR 901.6828 million, about 12.3% higher than the FBF reserve of IDR 803.0007 million. Crucially, the total uncertainty measure Unc for FBF (771.8853) is approximately 14.7% lower than that of FCL (904.7808). This lower uncertainty indicates that, within the fuzzy uncertainty framework adopted in this study, the FBF method produces reserve estimates with a narrower range of vagueness than the FCL method.

Table 9 Sensitivity Analysis and Total Uncertainty

Parameters	FCL	FBF
β	$E_\beta\left(\hat{\hat{R}}^{FCL}\right)$	$E_\beta\left(\hat{\hat{R}}^{FBF}\right)$
0.1	539.7705	494.2465
0.2	630.2486	571.4351
0.3	720.7266	648.6236
0.4	811.2047	725.8121
0.5	901.6828	803.0007
0.6	992.1609	880.1892
0.7	1082.6390	957.3777
0.8	1173.1171	1034.5662
0.9	1263.5952	1111.7548
	$Unc\left(\sum_{i=1}^{I} \hat{\hat{C}}_{i,J}^{FCL} \vert \mathcal{O}_I\right) = 904.7808$	$Unc\left(\sum_{i=1}^{I} \hat{\hat{C}}_{i,J}^{FBF} \vert \mathcal{O}_I\right) = 771.8853$

The collective results demonstrate the distinct operational characteristics of the two fuzzy approaches. The FCL method relies solely on historical development patterns and provides a transparent, data-driven estimate that is valuable as a conservative benchmark or in situations where reliable prior information is absent. However, this comes at the cost of higher overall uncertainty and greater sensitivity to the choice of risk parameter β.

Conversely, the FBF method leverages prior information to anchor its predictions. This results in more stable central reserve estimates and lower overall uncertainty within the adopted fuzzy framework. These features make FBF particularly advantageous when credible prior knowledge is available.

It should be noted that the comparison conducted in this study is based on model-derived reserve estimates and fuzzy uncertainty measures. Since actual ultimate claim outcomes were not available for validation, the results should be interpreted as a comparison of reserve behavior and uncertainty representation rather than predictive accuracy.

V. CONCLUSIONS AND SUGGESTIONS

This study provides an empirical comparison of the Fuzzy Chain Ladder (FCL) and Fuzzy Bornhuetter-Ferguson (FBF) methods for liability claim reserving. Based on the analyzed liability insurance portfolio, the results indicate a clear trade-off between the two approaches. The FCL method yields higher and more volatile reserve estimates, reflecting uncertainty contained in historical claims development patterns. In contrast, the FBF method produces lower and more stable estimates through the incorporation of prior information. In the analyzed dataset, FBF also exhibits lower total uncertainty and an asymmetric uncertainty structure within the adopted fuzzy uncertainty framework. Therefore, the choice of method depends on the specific reserving context. FCL may serve as a transparent, data-driven benchmark when reliable prior information is unavailable, whereas FBF may be advantageous when credible external estimates are available. However, the present study does not evaluate predictive accuracy against realized ultimate claims. Therefore, the conclusions are restricted to differences in reserve estimates and uncertainty characteristics under the fuzzy reserving framework.

A limitation of this study is that the analysis was conducted using a single 6×6 run-off triangle derived from one liability insurance portfolio. Therefore, the findings should be interpreted as specific to the analyzed portfolio rather than universally applicable. Future research should evaluate the performance of the FCL and FBF methods using multiple portfolios, different lines of business, larger run-off triangles, or simulation-based datasets to further assess the robustness and generalizability of the results.

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