ACCURACY INVESTIGATION OF THE THREE-POINT RESECTION METHOD THROUGH THE DISTRIBUTION OF CONTROL POINTS ACROSS FOUR QUADRANTS

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ABSTRACT

The three-point resection method is a valuable and effective technique in surveying that provides accurate and efficient solutions for determining the position of a resected point (point of unknown location). This paper presents a simple and innovative approach to determining the accuracy of the three-point resection problem in surveying. The method involves distributing control points (points of known locations) across four quadrants and computing the coordinates of the resected point (P) several times in order to see which quadrant provides better accuracy. The study investigates the impact of the positions of the control points, either in one quadrant or a combination of quadrants, on the accuracy of the resected point, which is a new contribution to the existing literature. The primary objective of this article is to explore the influence of the distribution of control points in different quadrants on the accuracy of the resected point. Furthermore, the study aims to determine the optimal positions of the three points in terms of their positions in one quadrant or their positions in a combination of quadrants. The relationship between the relative positions of the resected point and other control points, and the accuracy of the resected point is also examined in detail. The results of this study show that the relative positions of the control points and the resected point significantly impact the accuracy of the resected point. The paper concludes by defining the positions of control points distributed across quadrants that result in the best accuracy of the resected point.

Keyword: Three-Point Resection, Surveying, Triangulation, Resected point, Resection and Intersection.

Introduction

The three-point resection method is a widely used technique in surveying engineering that enables surveyors to determine the coordinates of a resected point. The method involves measuring the angles and distances between the resected point and three points with known coordinates (control points). One of the key advantages of this method is its ability to provide highly accurate and precise results, even in challenging environments where direct measurements may be difficult or impossible [1]. Another benefit of the three-point resection method is its cost-effectiveness and time efficiency. Unlike other surveying techniques that may require more complex equipment or a larger number of control points, this method requires only basic surveying equipment and three control points [2]. This makes it an ideal technique for surveyors working in remote or hard-to-reach locations where sophisticated equipment may not be available. In addition to its applications in surveying, the three-point resection method is also useful in photogrammetry. In this field, it is used to determine the position of an aerial photograph's perspective center (resected point) relative to the ground. This is a critical step in creating accurate photogrammetric maps and generating digital elevation models from aerial photographs. While the three-point resection method is an effective tool, it does have its limitations. For example, it may not be effective when the control points and resected points lie on the circumference of a circle, or when the control points lie on a straight line. However, these limitations can often be overcome by using additional control points or alternative surveying methods (refer to the THREE POINT RESECTION PROBLEM from Surveying Engineering Department at Ferris State University).

In recent decades, several analytical methods have been developed, such as the Collins method [3], [4], the Kaestner-Burkhard method [3], [4], and the Cassini method [5], all of which determine the location of the resected point (p) using Cartesian coordinates. These methods use different geometrical relations between the position of four points (A, B, C, and P) and the angles. An alternative method for resection is the Tienstra method, also known as the barycentric method [6]. This method provides the position of a point (P) in terms of barycentric coordinates, which are a linear combination of the station's coordinates [7]. A variety of analytical and graphical solutions have been proposed for the three-point resection problem, and these have been extensively discussed in previous articles and books e.g., [2] [8] [9] [10]–[15]. As a result, surveyors and engineers can easily access the necessary information and guidance to perform this technique effectively.

The accuracy of the three-point resection method can be affected by various factors, such as the position and distribution of the control points, the quality of the instruments used, and the errors introduced during the observation and calculation process. Hence, it is crucial to investigate these factors in order to enhance the reliability and efficiency of the three-point resection method. In this article, our main objective is to investigate the impact of the position of the three control points on the accuracy of the three-point resection method. Specifically, we will examine the positions of the three control points in a single quadrants or a combination of quadrants, and compare the obtained coordinates with the actual coordinates of the resected point. We will also explore whether there is a relationship between the position of the three control points and the accuracy of the resected point, and how this relationship can be quantified and optimized. Our study is unique in that previous articles and books on the subject did not address this question directly, and only provided general guidelines and procedures for using the three-point resection method. By providing a comprehensive understanding of the impact of control point locations on the accuracy of the three-point resection method, our research aims to help surveyors make better decisions when using this technique and to improve the overall quality and efficiency of surveying. To achieve our objectives, we have employed a combination of theoretical analysis, computer simulation, and field experiments. Firstly, we conducted our own field experiment by measuring control points and performing all the necessary surveying observations for the three-point resection method (see the instruments we used at Figure 1) Secondly, we developed a MATLAB script for our mathematical model of the three-point resection method. Lastly, we validated and discussed the results obtained from the three-point resection method with the MATLAB script using the actual coordinates obtained from the surveying measurements. Figure 2 shows the flowchart of our investigation strategy for the three-point resection method, which involves multiple stages and methods, and aims to provide a comprehensive and systematic analysis of the factors that affect the accuracy of this method.

(a) (b)

Figure 1. Instruments used in the surveying works (a) GPS Stonex S9 III. (b) Total Station Leica TS02 PLUS.

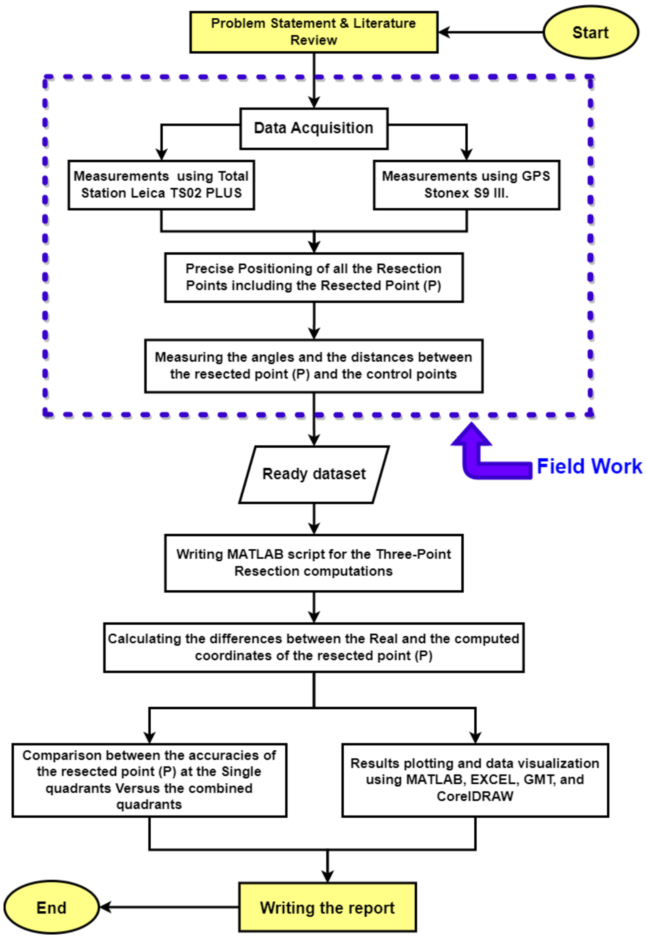

Figure 2. Flowchart of the investigation strategy for the three-point resection method accuracy.

The resection method

In surveying and photogrammetry, there are three commonly used resection methods: the space resection on photogrammetry, the two-point resection method in surveying engineering (free station), and the three-point resection method. The space resection on photogrammetry involves determining the position and orientation of a camera in space using multiple images, the two-point resection method is useful for determining the position of inaccessible points or features in the field, and the three-point resection method is widely used in engineering and construction projects to locate objects such as buildings, bridges, and other infrastructure. Each method has its unique advantages and limitations, and the choice of technique depends on the specific requirements of the project. The three-point resection method is particularly useful for determining the location and orientation of a resected point, making it an important tool in the field of engineering and construction [16][17][18].

Space resection on photogrammetry

The space resection method is used in photogrammetry to determine the camera's position and orientation in space, using control points and image coordinates to calculate the exterior orientation parameters (EOP) of the camera. The EOP are crucial for generating accurate 3D models and maps. This method involves an optimization model that minimizes the sum of squared residuals between the observed and calculated image coordinates, which represents the error in the measurements. The objective of the model is to minimize these errors, resulting in a more accurate determination of the camera's position and orientation. The space resection method is described as an optimization model with an objective function to minimize the sum of squared residuals [19] [20]. The model can be expressed as follows:

$$X = (f)\left(\frac{r_{11}(X_A - X_L) + r_{12}(Y_A - Y_L) + r_{13}(Z_A - Z_L)}{r_{31}(X_A - X_L) + r_{32}(Y_A - Y_L) + r_{33}(Z_A - Z_L)}\right) \quad (1)$$

$$y = (f)\left(\frac{r_{21}(X_A - X_L) + r_{22}(Y_A - Y_L) + r_{23}(Z_A - Z_L)}{r_{31}(X_A - X_L) + r_{32}(Y_A - Y_L) + r_{33}(Z_A - Z_L)}\right) \quad (2)$$

Where (X_A, Y_A, Z_A) are the terrestrial coordinates, (x, y) are the image coordinates, (X_L, Y_L, Z_L) are the coordinates of the exposure station or the camera, (f) is The focal length of the camera, and (r_{ij}) is the rotation matrix elements, if the value of camera's focal length and the coordinates of the ground point are known, then the Equation (x, y) becomes a function of the three exposure station elements (X_L, Y_L, Z_L) and the rotation angle elements (ω, ϕ, k), then By distribution the equations in a Taylor series, it will obtain linear equations that can be solved by numerical methods and it can be solved by least square method as well.

Two-points resection method

The two-point resection method, also known as the free station method, is used in surveying to determine the position of a new control point on a network. This method involves taking measurements and observations from the new point to two or more control points on the network. Two types of resection can be performed: angular resection and distance resection. Both of these methods are useful in situations where it is not possible to set up a control point directly on the site of interest. The free station points obtained using the two-point resection method can be used for a variety of purposes, including setting up temporary benchmarks, establishing reference points for construction projects, and creating accurate maps and plans of the area [16].

Angular resection

Angular resection is a surveying technique that determines the location of a point on the Earth's surface by measuring angles from it to existing control points. At least three control points are required, and the angles between the resected point and each control point are measured using a theodolite or similar instrument. Trigonometric principles are then used to calculate the position of the resected point relative to the control points. This method is versatile and can be performed in various settings, including urban areas where access to control points may be limited. Angular resections are used to determine the position of objects such as buildings and towers that may be difficult to measure directly. This method is a valuable tool for surveyors and engineers in a wide range of applications. The positions of the resected point in a three-point resection are illustrated in Figure 3 [18] (refer to Topic 6: Angle measurement: Intersection and resection).

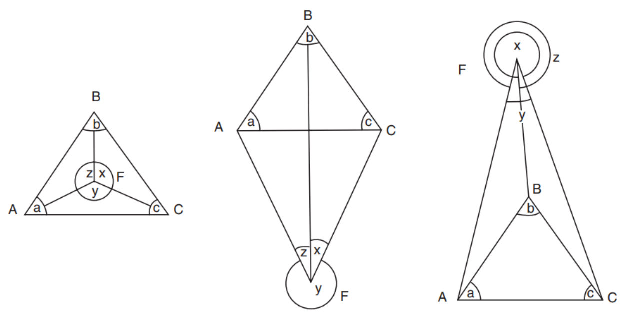

Figure 3. The three cases of the angular resection [18].

Methodology

The three-point resection method involves measuring the angles between the control points and the resected point, and then using these angles to calculate the coordinates of the resected point. This method is commonly used in navigation, mapping, and construction, and requires three control points to determine the position of the resected point [16] [18].

The three-point resection method has three cases, as shown in Figure 3, and the location of the resected point depends on the position of the control points and the polygon formed by them. The resected point can either be in the middle of the polygon or in one corner of the polygon. This information is from a source cited as [17]. The article focuses on the case where the resected point is in the corner of the polygon, as shown in Figure 4.

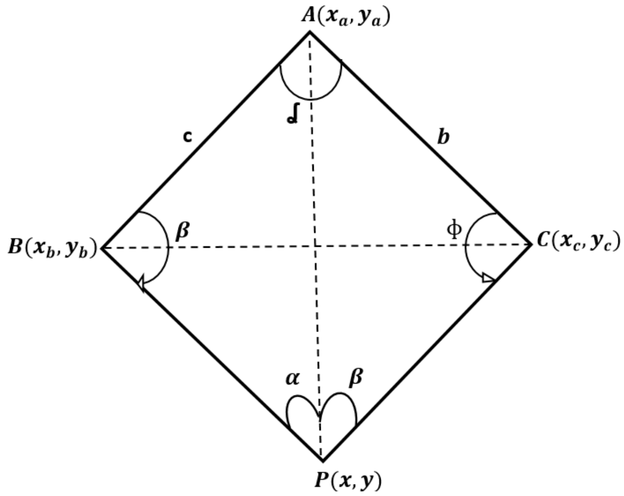

Figure 4. The positions of resected point P and control points (A, B, and C) [7].

Where: P represents the resected point, which is the resected point whose coordinates we are trying to determine. A, B, and C are the control points, which are the control points whose coordinates are already established. α and β are the angles between the control points A and B, B and C, respectively, measured from the resected point P. ϕ, β, and ʆ are the angles that will be calculated mathematically using the following formulas. b and c are the distances between the three control points A, B, and C, and they will also be calculated mathematically using the formulas.

There have been many articles produced on the solution of the three-point problem using different methods. In this paper, we used the solution proposed by Kaestner-Burkhardt Method [7] and found that it gives high accuracy compared to other solutions. The first step is to calculate the angles, given that points A, B, and C are already known. The angle (ʆ) can be calculated by finding the difference in deflection of the two lines.
To compute the assisting angles ɣ and ∅, we first need to remember that the sum of the interior angles of a polygon is equal to 360 degrees. This can be expressed mathematically as (n – 2)*180 degrees, where n is the number of sides in the polygon [17] [7].

$$\alpha + \text{ɣ} + \beta + \text{ʆ} + \emptyset = 360 \qquad (3)$$

Rearrange Equation (3) to solve the angle ɣ:

$$\text{ɣ} = 360 - (\alpha + \beta + \text{ʆ}) - \emptyset \qquad (4)$$

Since the interior angles α, ʆ, and β are known, we need to calculate the angles ɣ and Ø. We can do this using the following equations:

$$ɣ = R - Ø \quad (5)$$

where R is a constant angle.

Assume that:

$$K_1 = \frac{b}{c} * \frac{\sin(\alpha)}{\sin(\beta)} = \frac{\sin(ɣ)}{\sin(Ø)} \quad (6)$$

Or we can say:

$$K_2 = \frac{c}{b} * \frac{\sin(\beta)}{\sin(\alpha)} = \frac{\sin(Ø)}{\sin(ɣ)} \quad (7)$$

So we conclude that:

$$K2 = \frac{1}{K1} \quad (8)$$

Calculating the distance of c and b between the control points by:

$$c = \sqrt{(A_x - B_x)^2 + (A_y - B_y)^2} \quad (9)$$

$$b = \sqrt{(A_x - B_x)^2 + (A_y - B_y)^2} \quad (10)$$

Using Equations (6), (7), (9), and (10), we can calculate the angles Ø and ɣ as follows:

Compute Ø

$$\cot(Ø) = \frac{K_1 + \cos(R)}{\sin(R)} \quad (11)$$

$$\cot(ɣ) = \frac{K_2 + \cos(R)}{\sin(R)} \quad (12)$$

In Figure 4, by taking the triangle (APC), we can solve for the angle ɸ on point C by using Equation (9) and finding the deflection of the line (CP) from the deflection of the line (CA) (known deflection). Then, we can calculate the coordinates of the resected point (P) using the following equations [7]:

$$\Delta E = L * \sin(\alpha) \quad (13)$$

$$\Delta N = L * \cos(\alpha) \quad (14)$$

Here, ΔE is the difference in Easting coordinates between resected point (P) and control point (C), and ΔN is the difference in Northing coordinates between resected point (P) and control point (C).

Therefore, we can compute the coordinates of the resected point (P) using the following equations:

$$E_P = E_B + L * sin(\alpha) \quad (15)$$

$$N_P = N_B + L * cos(\alpha) \quad (16)$$

Alternatively, we can compute the coordinates of the resected point (P) by using the triangle (APB) in Figure 4 and finding the angle Ø using equation (9). Then, the final equations for computing the resected point will be as follows:

$$\Delta E = L * \sin(Ø) \quad (17)$$

$$\Delta N = L * \cos(Ø) \quad (18)$$

Where L is the distance between resected point (P) and control point (B), ΔE is the difference in Easting coordinates between resected point (P) and control point (B), and ΔN is the difference in Northing coordinates between resected point (P) and control point (B).

Alternatively, we can also compute the coordinates of the resected point (P) using the following equations:

$$E_P = E_B + L * sin(Ø) \quad (19)$$

$$N_P = N_B + L * cos(Ø) \quad (20)$$

These equations can be used to verify the correctness of the previous computations.
In this study, 21 stations were measured for our resection competitions. To obtain the necessary measurements, a Total station instrument model of Leica TS02 PLUS as shown in Figure 1b was used to measure the angles and distances between the control points. Also, Figure 1a multi-frequency GNSS receiver model of Stonex S9 III shown in a was utilized during the observation sessions to eliminate most of the ionospheric effects and also in order to measure the real coordinates of all the control points and also the resected point (P).
All 21 control points were measured with a high-precision GPS-RTK instrument to ensure accurate and precise data collection. The study was meticulously executed to ensure reliable and valid results. Control points were divided into four quadrants shown in Figure 5, providing an overview of the study area's spatial distribution for a deeper analysis of the data.

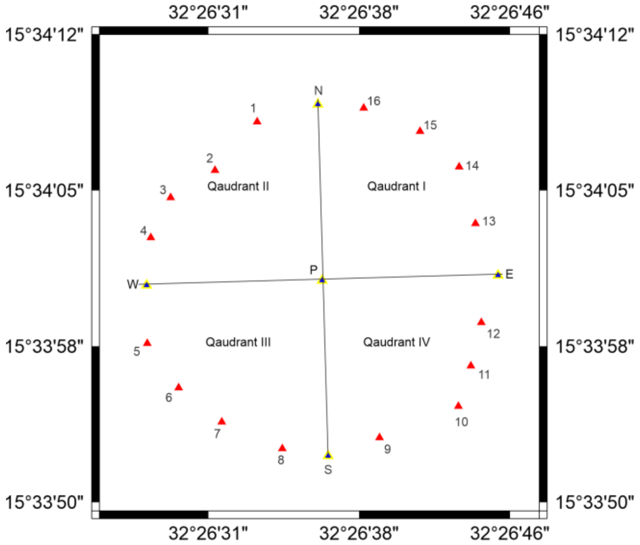

Figure 5. Distribution of the control points around the resected point across four-quadrants.

Figure 5 provides a clear representation of the distribution of the control points in each quadrant. It is evident that four control points have been distributed in every quadrant except for the control points, which are located in the vertical and horizontal axes. To ensure a comprehensive understanding of the control points, they have been divided into eight sectors and two groups, with each group containing four sectors.

The first group, Group A, is represented by the first quadrant, second quadrant, third quadrant, and fourth quadrant. On the other hand, Group Two consists of combinations between two quadrants. These combinations include the first combination between the first and second quadrants, which contains control points named (E, 13, 14, 15, 16, N, 1, 2, 3, 4, W). The second combination comprises the second and third quadrants, which contain control points named (N, 1, 2, 3, 4, W, 5, 6, 7, 8, S). The third combination is between the first and second quadrants and contains control points named (W, 5, 6, 7, 8, S, 9, 10, 11, 12, E). Lastly, the fourth combination is between the first and second quadrants and contains control points named (S, 9, 10, 11, 12, E, 13, 14, 15, 16, N). The procedure for the three-point accuracy investigation is shown in Figure 2, which illustrates the flowchart for the investigation strategy.

Result and Discussion

The study compared the coordinates of the GPS-RTK and the three-point resection method in determining the accuracy of the resection point in single and combined quadrants. The results showed high accuracy in all four quadrants and were presented in tables. The coordinates of the resected point were determined twice and compared to provide insights into the effectiveness of the three-point resection method.

Most of the results have been presented with a difference of three decimal digits, indicating a high level of accuracy. However, there have been some exceptions to this trend. In the first quadrant, two cases have resulted in differences greater than 1 meter in the northern coordinate compared to the actual coordinates of the resected point (P). These cases involved the combination (the triangle) of the (S, 11, 12) control points with the resected point (P) and the combination of (S, 12, E) control points with the resected point (P).

In the second quadrant, there has been one case where the differences were greater than 1 meter in the northern coordinate and over 2 decimeters in the eastern coordinate. This case involved the combination (the triangle) between the (W, 5, 6) control points with the resected point (P).

In the third quadrant, there have been three cases where the control points have given rise to significant differences. Taking the combination of points (2, 3, 4) with the resected point (P) has resulted in differences greater than 2 decimeters in the northern coordinate. The combination of points (3, 4, W) with the resected point (P) has resulted in differences greater than 4 decimeters in the northern coordinate. Finally, the combination between points (N, 1, 3) with the resected point (P) has resulted in differences greater than 5 decimeters in the eastern coordinate.

Finally, the case in the fourth quadrant combined from the points (E, 15, 13) with the resected point (P) has resulted in differences greater than 1 meter in both the northern and eastern coordinates. Nonetheless, the results provide valuable insight into the effectiveness of the three-point resection method and its limitations.

The results of the four single-quadrants:

The study analyzed the results of four single quadrants: the first, second, third, and fourth quadrants, to determine the accuracy of the collected data. In most cases, the accuracy reached millimeters and centimeters, but there were a few exceptions. Specifically, two cases in the first quadrant, one case in the second quadrant, three cases in the third quadrant, and one case in the fourth quadrant exhibited some level of inaccuracy.

Table 1. The differences between the real and the computed coordinates of the resected point (P) in the first quadrant.

No.	Combination	ΔE	ΔN
1	<>P 10,9,S	0.001	0.001
2	<>P 11,10,S	-0.002	-0.003
3	<> P 12,11,S	-0.030	1.280
4	<> P E,12,S	-0.035	1.273
5	<> P 11,10,9	-0.001	-0.003
6	<> P 12,11,9	0.010	-0.070
7	<> P E,12,9	-0.022	0.053
8	<> P 12,11,10	-0.030	-0.028
9	<> P E ,12,10	0.060	0.013
10	<> P E,12,11	-0.070	0.053

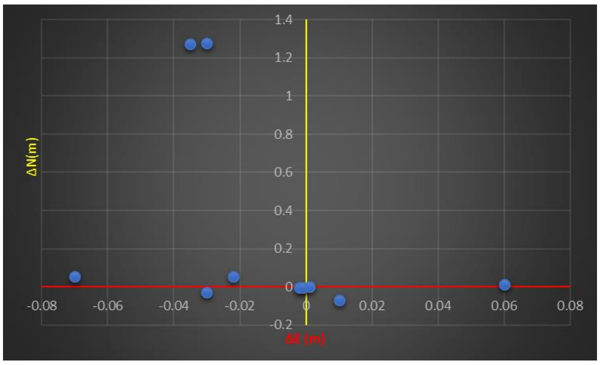

Figure 6. The accuracy of the resected point (P) in the first quadrant.

Table 1 shows that there are two cases in the first quadrant where the differences in the northern coordinate compared to the real coordinates of the resected point (P) are greater than 1 meter. These cases are: the combination of the (12, 11, S) control points with the resected point (P), which results in a difference of -0.030 meters in the eastern coordinates and 1.280 meters in the northern coordinates; and the combination of the (E, 12, S) control points with the resected point (P), which results in a difference of -0.035 meters in the eastern coordinates and 1.273 meters in the northern coordinates. As seen in Table 1 and Figure 6, some of the differences in the first quadrant are in millimeters, while most of them are in centimeters.

Table 2. The differences between the real and the computed coordinates of the resected point (P) in the second quadrant.

No.	Combination	ΔE	ΔN
1	<>P 5,6,7	0.010	0.029
2	<>P 5,6,8	0.000	0.000
3	<>P 6,7,8	0.080	0.010
4	<>P 7,8,S	-0.003	0.001
5	<>P 6,8,S	-0.001	0.000
6	<>P 5,6,S	-0.012	0.011
7	<>P W,5,6	-0.220	-1.030
8	<>P W,6,7	0.002	0.001
9	<>P W,7,8	0.008	0.001
10	<>P W,8,S	0.004	0.000

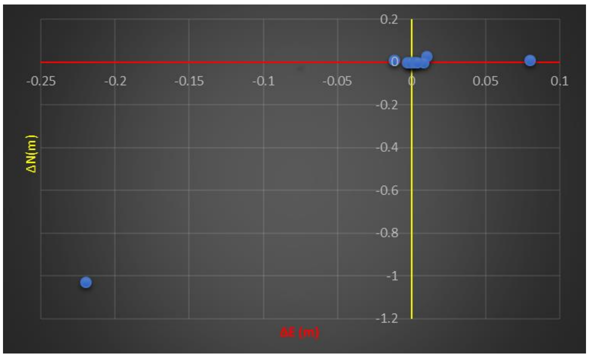

Figure 7. The accuracy of the resected point (P) in the second quadrant.

Table 2 shows that there is one case in the second quadrant where the differences are about -1.030 meters in the northern coordinate and -0.220 meters in the eastern coordinates. This case occurs when combining the (W, 5, 6) control points with the resected point (P). As shown in Table 2 and Figure 7, some of the differences in the second quadrant are in centimeters, while most of them are in millimeters.

Table 3. The differences between the real and the computed coordinates of the resected point (P) in the third quadrant.

No.	Combination	ΔE	ΔN
1	<>P N,1,2	0.004	0.002
2	<>P 1,2,3	0.000	0.000
3	<>P 2,3,4	0.056	0.230
4	<>P 3,4,W	0.011	-0.460
5	<>P N,3,4	0.000	0.001
6	<>P N,1,3	-0.580	-0.025
7	<>P 1,2,W	-0.001	0.010
8	<>P 1,4,W	-0.001	0.002
9	<>P N,2,4	0.000	-0.002
10	<>P 2,4,W	0.000	-0.010

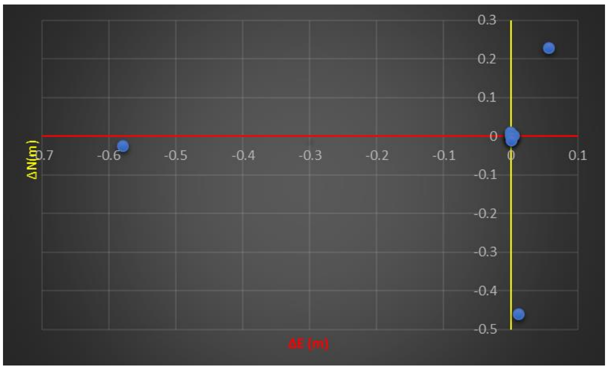

Figure 8. The accuracy of the resected point (P) in the third quadrant.

Table 3 shows that there are three cases for the control points located in the third quadrant. When combining points (2, 3, 4) with the resected point (P), the difference in the northern coordinate is about 0.230 meters. When combining points (3, 4, W) with the resected point (P), the difference in the northern coordinate is about -0.460 meters, and the difference in the eastern coordinate is 0.011 meters. Additionally, when combining points (N, 1, 3) with the resected point (P), the difference in the eastern coordinate is about -0.580 meters, and the difference in the northern coordinate is -0.025 meters (See Figure 8).

Table 4. The differences between the real and the computed coordinates of the resected point (P) in the fourth quadrant.

No.	Combination	ΔE	ΔN
1	<> P N,16,15	0.003	-0.003
2	<> P 16,15,14	0.006	-0.002
3	<> P 15,14,13	-0.016	0.011
4	<> P 14,13,E	0.003	-0.003
5	<>P 16,15,13	0.001	0.000
6	<>P N,15,14	-0.489	-0.010
7	<>P 16,13,E	0.141	-0.034
8	<>P 15,13,E	-0.068	0.053
9	<>P N,13,E	0.010	0.000
10	<>p N,14,E	0.010	0.000

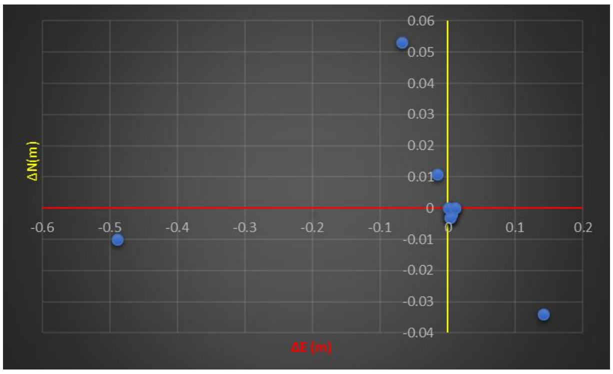

Figure 9. The accuracy of the resected point (P) in the fourth quadrant.

Finally, Table 4 shows the case in the fourth quadrant where the combination of points (N, 15, 14) with the resected point (P) results in differences of about -0.489 meters in the northern coordinates and -0.010 meters in the eastern coordinates (See Figure 9).

The results of the Combination quadrants

The Combination quadrants are points located at the intersection of the first and second, second and third, third and fourth, and fourth and first quadrants. These quadrants have a high level of accuracy, measured in millimeters, due to the convergence of several factors. These factors include the position of the point relative to the quadrants, the angle of the point relative to the quadrants, and the degree of intersection between the quadrants. The tables and graphs show that the combination quadrants have superior accuracy compared to the single quadrants, and the precision of the points in the combination quadrants is measured in millimeters.

Table 5. The differences between the real and the computed coordinates of the resected point (P) at the combination of the first and second quadrants.

NO	Combination	ΔE	ΔN
1	<>P 1,N,16	-0.013	-0.002
2	<>P 1,15,14	0.000	0.001
3	<>P 1,14,13	0.001	0.002
4	<>P 1,13,E	0.000	-0.001
5	<>P 2,16,15	0.000	0.000
6	<>P 2,15,14	0.000	0.000
7	<>P 2,14,13	0.000	0.000
8	<>P 3,16,15	0.000	0.000
9	<>P 3,14,13	0.000	0.000
10	<>P 4,13,E	0.000	0.000

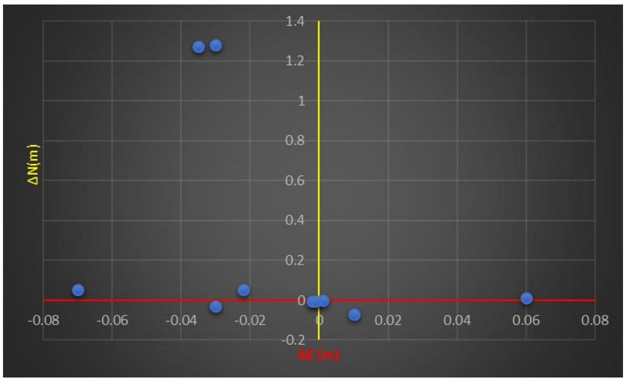

Figure 10. The accuracy of the resected point (P) at the combination of the first and second quadrants.

Table 5 and Figure 10 display the results for the combination of the first and second quadrants, where all the cases have millimeter accuracy except for the triangle of (1, N,16), which shows differences of -0.013 meters in the eastern coordinates and -0.002 meters in the northern coordinates.

Table 6. The differences between the real and the computed coordinates of the resected point (P) at the combination of the second and third quadrants.

No	Combination	ΔE	ΔN
1	<>P 13,12,11	0.001	0.001
2	<>P 13,11,10	0.002	0.001
3	<>P 13,10,9	0.000	0.001
4	<>P 13,9,S	0.006	0.002
5	<>P 14,12,11	0.001	0.001
6	<>P 14,11,15	0.000	0.003
7	<>P 15 ,12,11	0.003	0.002
8	<> p 15,10,9	-0.001	0.000
9	<>P 16,11,10	0.000	0.001
10	<>P 16,9,S	0.000	0.000

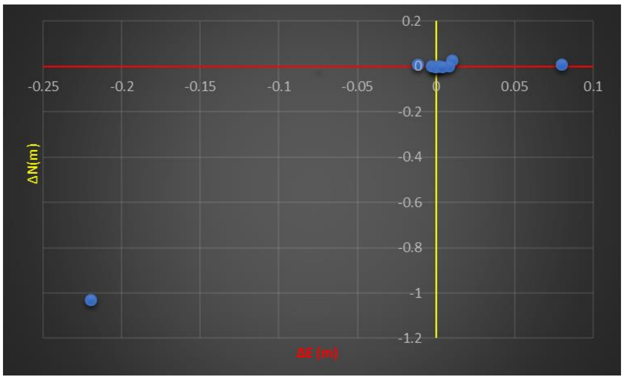

Figure 11. The accuracy of the resected point (P) at the combination of the second and third quadrants.

Table 6 and Figure 11 show the results for the same combination of the first and second quadrants, where all the cases have millimeter accuracy. This indicates that these results are more accurate than the previous graphs.

Table 7. The differences between the real and the computed coordinates of the resected point (P) at the combination of the third and fourth quadrants.

No	Combination	ΔE	ΔN
1	<> P 9,8,7	0.009	-0.005
2	<> P 9,7,6	0.005	-0.002
3	<> P 9,6,5	0.001	0.000
4	<> P 9,5,W	0.000	0.000
5	<> P10,8,7	-0.002	0.000
6	<> P 10,7,6	0.000	0.000
7	<> P 11,8,7	0.001	0.000
8	<> P 11,6,5	0.001	0.003
9	<> P 12,8,7	0.001	0.000
10	<> P 12,6,5	0.000	0.000

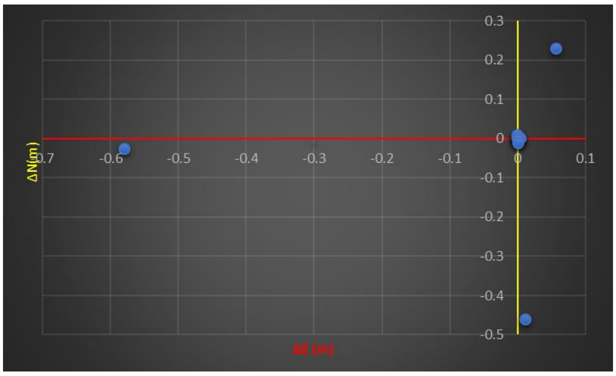

Figure 12. The accuracy of the resected point (P) at the combination of the third and fourth quadrants.

Similarly, Figure 11, Table 7 and Figure 12 also display the results for the combination of the first and second quadrants, where all the cases have millimeter accuracy.

Table 8. The differences between the real and the computed coordinates of the resected point (P) at the combination of the fourth and first quadrants.

No	Combination	ΔE	ΔN
1	<> P 5,4,3	0.000	0.001
2	<> P5,3,2	-0.010	0.000
3	<> P 5,2,1	-0.002	0.001
4	<>p 5,1,N	0.000	0.000
5	<>P 6,3,2	0.003	0.000
6	<>P 6,2,N	0.000	0.000

7	<> P 7,3,2	0.000	0.010
8	<> P7,2,N	0.001	0.000
9	<> P 8,3,2	0.000	0.001
10	<>P 8,2,N	0.000	0.000

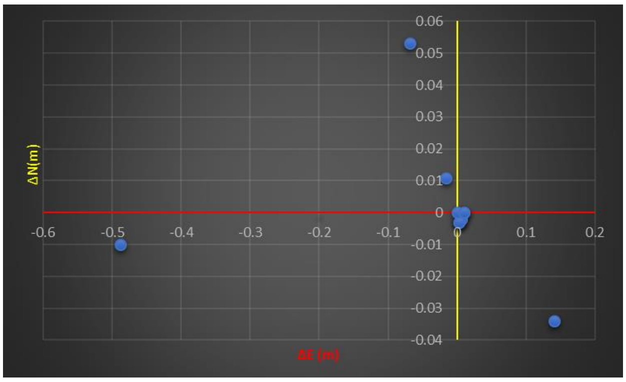

Figure 13. The accuracy of the resected point (P) at the combination of the fourth and first quadrants.

Table 8 and Figure 12 show the results for the combination of the first and second quadrants, where all the cases have millimeter accuracy except for the triangle of (5,3,2), which shows differences of -0.010 meters in the northern coordinate and no differences in the eastern coordinates. Additionally, the triangle of (7,3,2) shows differences of -0.010 meters in the eastern coordinate and no differences in the northern coordinates.

Conclusion

In this paper, a new accuracy investigation strategy of the three-point resection method has been presented. The proposed strategy involves distributing control points across four quadrants and computing the coordinates of the resected point (P) multiple times to evaluate which quadrant provides better accuracy. The study provides new insights into the problem by analyzing the impact of the positions of the control points, either in one quadrant or a combination of quadrants, on the accuracy of the resected point.

The three-point resection procedure was performed using several control points (control points), and the differences between the real coordinates of the resected point and its computed coordinates from different quadrants were analyzed. Based on the results, we concluded that there is a correlation between the positions of the control points and the accuracy of the resected point (P). The results presented above show that the resected point's accuracy is higher when the control points are positioned in a combination of quadrants rather than in only one quadrant, as used in this study. This observation highlights the importance of carefully selecting the control points' positions to achieve the desired accuracy level in the three-point resection solution.

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Appendix A

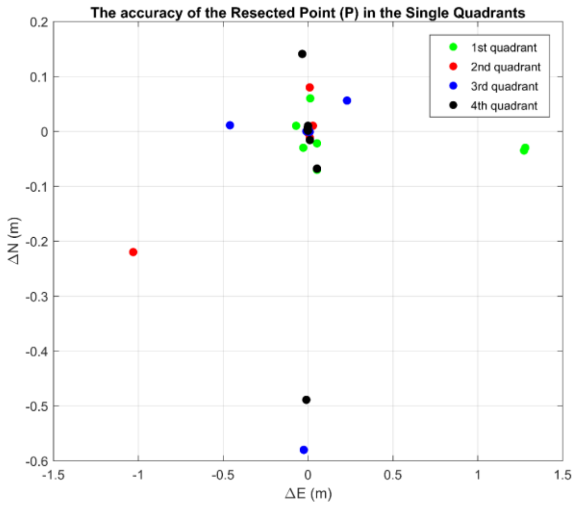

Figure A1. The accuracy of the resected point (P) at the single quadrants.

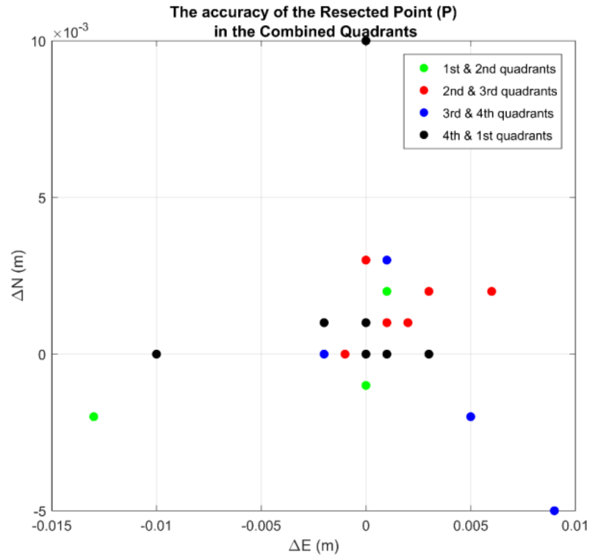

Figure A2. The accuracy of the resected point (P) at the single quadrants.

Appendix B

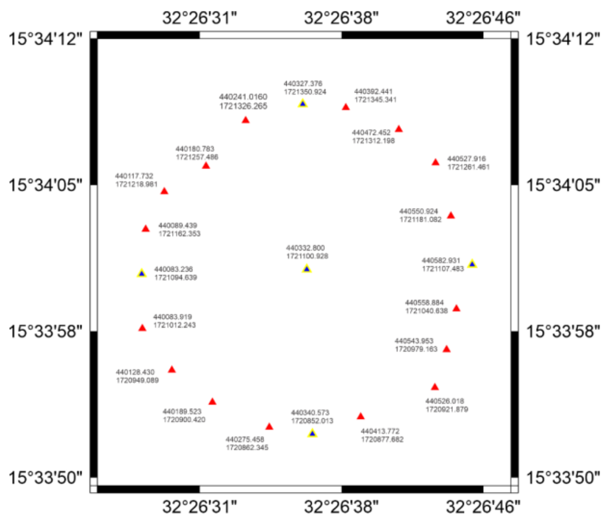

Figure B1. The coordinates of the resection points.

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